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THE RADIO BACKGROUND OF RADAR

BY

HENRY GUERLAC [1]

Radio detection devices using the pulse-echo principle were developed independently and almost simultaneously during the 1930's by a number of the great powers. In 1939 closely guarded secret programs were in various stages of advancement in Great Britain, France, Germany, Canada and the United States. Russia, China, Japan and Italy were at that time without the equipment and seem to have acquired it after the outbreak of war, by capture and by disclosures from their allies. In France and Germany, radar was developed in commercial laboratories under military supervision, whereas in Great Britain and the United States the pioneer work was done by the armed services.

Such a duplication of effort will surprise only those who cling to a Hero Theory of scientific progress and demand for each discovery a single putative inventor; or those who are unaware of the frequency—one is tempted to write, the regularity—with which such parallelisms are encountered in scientific work. The duplications are by no means rare in pure science; but they are much more common in applied research, where a significant discovery, simultaneously taken up by many hands, can spawn a numerous progeny of practical applications and useful embodiments. The chances of duplication are better in the present century than in the last, for in the final analysis they depend only upon the scope and intensity of the research effort. It has been well said that the greatest invention of the nineteenth century was the art of inventing; and this is an art which the twentieth century has greatly extended and perfected by widespread technical education, well-financed team research, generous budgets, and well-equipped laboratories. The element of chance in scientific progress (which includes also the chance of discoveries by isolated individuals being lost to society) is steadily dimin-

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(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

ishing, and the discontinuities in the upward curve of technical progress, though they are still present, are less pronounced.

Each striking instance of parallel and independent discovery raises a number of fundamental historical questions. When, moreover, a real burgeoning takes place we are led to ask what the conditions were that favored it. The main influences are obvious. First of all, when a serious effort is put behind a development, as in the case of radar, it is evident that this development satisfied a clear and urgent need. Second, since the principle of *nihil ex nihilo* applies quite as surely to the history of science as to science itself, it is obvious that some key principles or central ideas in pure science must have been the point of departure. Lastly, if success is finally attained, it is certain that the state of the art, that is to say the perfection of engineering skills in this and neighboring fields, must have reached a point where success was fairly well assured.

It is proposed in this paper first to analyze the military need for radio detection equipment, and then to explore the background of scientific ideas and technical progress that together made possible or perhaps inevitable the development of radar before World War II. Conditions were definitely propitious in the period 1930–1939; so much so, in fact, that, although there were important adumbrations long before, it is hard to imagine a successful development much before that time.

THE REPLY TO AIR POWER

Radar was at first envisaged solely as a defensive weapon against hostile aircraft. The first types to be designed were long-range, high-power sets to give early warning against attacks by enemy planes, or sets with shorter range to provide accurate information for the direction of searchlights or the pointing of anti-aircraft guns. Viewed in historical perspective, radar will probably appear as the reply, the countermeasure, to the threat of air power.

Perhaps we are still too close in time to realize fully that the ten years before World War II were the years that introduced the age of air power. In 1930 we were still marvelling at Lindbergh's solo flight to Le Bourget. By 1940 a series of technological changes had made routine commercial transatlantic flights a certainty.

While commercial air transportation became a commonplace during that decade, cruising speeds rose steadily. In 1931, Post and Gattie flew around the world in eight days; while in July, 1939, Howard Hughes cut this time in half, and maintained an average flying speed of 206 mph. On the eve of World War II, the principal countries of the world could all point to two- or four-motored aircraft in routine service having cruising speeds in excess of 200 mph. There was no doubt that existing aircraft warning devices were hopelessly archaic and inadequate, and serious efforts were being directed towards finding a completely novel type of device.

AIRCRAFT DETECTION IN THE FIRST WORLD WAR

Even the circumscribed role of aircraft in the First World War gave rise to methods of detection. This was limited in practice to detecting the sound of an approaching plane and determining its direction; but methods were also proposed for detecting and amplifying electromagnetic radiation (on radio and infra-red frequencies) emanating from the plane.

Sound detection seems to have begun, on the Allied side at least, with the "orthophone," a simple binaural device used in the French Army in 1917. At roughly the same time an acoustical detector of the reflector type was produced in England by the Anti-Aircraft Section (under Dr. A. V. Hill) of the Munitions Invention Department (1).[2]

By 1936 acoustical detectors had reached a point where an error of only $\frac{1}{4}°$ in azimuth was obtained on sounds from a fixed source. For an airplane flying at moderate heights all sound locator manufacturers quoted a 2° accuracy. In certain cases the sound was even converted into visual information on the screen of a cathode-ray oscilloscope.

These devices gave no range information; their performance depended upon the wind, and they were quite unreliable on gusty days. The velocity of sound is so low, and the range was so short, that with the high speed of modern planes observers scarcely had time to alert the defenders; certainly there was not enough warning to send up intercepting fighters. The main function of this equipment was to direct searchlights.

While with the AEF in France in the first World War, Edwin H. Armstrong, then a Major in the Signal Corps, conceived the idea that perhaps the extremely short radio waves produced by the spark gaps of a gasoline aircraft engine could be used to detect enemy aircraft. To solve the difficulty of detecting and amplifying such short waves, Major Armstrong developed the first superheterodyne receiver, but it was never used in World War I for the purpose for which it was devised (2).

Almost simultaneously a second method of attack—the attempt to detect the infra-red radiation from airplane engines and exhaust gases—was adopted early in 1918. Master Signal Electrician Samuel O. Hoffman and a group of enlisted men working under Professor George Pegram at Columbia University attempted to detect objects at higher temperature than their background by means of a thermophile mounted in the focus of a parabolic mirror (3). With this equipment in the spring of 1918 men were detected at 600 ft. In August 1918 an apparatus of this kind was sent overseas, but tests in France proved unsuccessful. Meanwhile thermal equipment specially modified to detect aircraft was tested at Langley Field, Va., where ranges from 4000 ft. to over a mile were reported but with complications from false indications caused by clouds.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

No further work on heat detection was carried on by the Army until 1926 when the Ordnance Department set up at the Frankford Arsenal a project for the "Investigation of Detection Devices using the Infra-Red Ray," under the direction of Captain William Sackville, C.A.C. This was a continuation of graduate research conducted with Lieutenant José R. Olivares, Philippine Scouts, U. S. Army, at M.I.T. (4).

The problem had been suggested by Professors Vannevar Bush and J. W. Barker, and under their direction the principal avenues of approach to the detection problem were reviewed—all, that is, except the use of *reflected* electromagnetic waves from a ground source. They rejected sound because of the time lag, short wave radio waves from the engine ignition system because of the ease of shielding, and for similar reasons visual observation of the exhaust flare, and focussed their attention on the infra-red energy from the engine exhaust. But experiments at M.I.T. and at the Frankford Arsenal were not much more successful than the closely similar Hoffman experiments.

DETECTION WITH REFLECTED ENERGY

The radar principle is divisible for purposes of analysis into two elements: detection by means of reflected radio energy (the echo principle) and the determination of range by means of carefully timed pulses (the pulse principle).

The earliest echo detection device was of course the human voice; a stentorian halloo from the bridge of a ship has often given the skipper of a fog-bound ship some notion of his proximity to land. But in the last century John Tyndall, the British physicist, at the request of the Elder Brethren of Trinity House, experimented with lights and with crude electrical contrivances and finally suggested the use of steam whistles to prevent ships from running ashore at night (5).

In 1912, after the sinking of the *Titanic*, Sir Hiram S. Maxim proposed to detect icebergs and other obstacles by sending out low-frequency sound waves, below the audible, and receiving the returning echo on a diaphragm (6). Maxim compared his method with Abbé Spallanzani's experimentally blinded bats (1794) which Maxim believed avoided obstacles by detecting the echo of low frequency sounds produced by their own wings (7).

THE REFLECTION OF RADIO WAVES

It is possible to detect invisible or distant objects by means of radio and infra-red energy only because such radiation has, like light, the property common to the whole spectrum of being reflected from obstacles. It is only because such waves can be focussed into directed beams of radiation that efficient utilization of energy is possible, and that detecting devices can act as direction finders or locators.

In the case of radio, this property is only readily observable for wave-

lengths a few meters in length or shorter. That is why attention was not drawn to reflection phenomena while interest was centered on broadcast frequencies, although these effects were prominent in the earliest experiments on radio waves.

In his great *Treatise on Electricity and Magnetism* (1873), James Clerk Maxwell brought under unified theoretical dominion the known facts concerning light, electricity and magnetism. The most striking consequence of Maxwell's electromagnetic theory of light was the prediction that there were other as yet undiscovered waves propagated through space with a velocity equal to that of light; that these are produced wherever oscillatory electric currents are set up, as for example in a discharging Leyden jar; and that they should be reflected from conducting surfaces and refracted by dielectrics according to the classical laws of geometrical optics. The existence of these radio waves was first experimentally verified in 1887–8, nearly ten years after Maxwell's death, by Heinrich Hertz. Hertz's experiments, once he had succeeded in generating and detecting the earliest known radio waves, were designed to prove that these waves did in truth have the resemblance to light, or as we say today, the quasi-optical properties, which Maxwell had predicted.

Hertz used wavelengths which made the optical properties easy to observe (**8**). In his first experiments his spark gap gave him waves 10 meters in length; subsequently with a much smaller oscillator he obtained a wavelength only 66 cm. His transmitter was placed in the focal line of a cylindrical metal reflector; his receiver consisted of a dipole in the focal line of a receiving reflector; the dipole was connected with a spark gap. With this equipment Hertz was able to detect radiation when the cylinders were separated by several meters; and with it he was able to duplicate many of the phenomena encountered in the study of light. He showed that shadows were cast when objects opaque to the radiation were interposed; he observed the phenomenon of diffraction; and he showed by rotating the receiving cylinder through 90° that the waves emitted by his oscillator were polarized. This was further confirmed by beautiful experiments in which he employed a grating of wires interposed between the transmitter and receiver: when the wires of the grating were at right angles to the linear radiator and to the receiving dipole the energy passed through; but when the screen was rotated through 90° the screen was opaque to the radiation. It likewise proved possible to refract electromagnetic waves. With a large prism of pitch cast in a wooden box Hertz was able to observe a refraction of as much as 22°; and from this result he was able to calculate the index of refraction of pitch for his waves to be 1.69, as compared with a value of 1.5 and 1.6 for similar substances when transmitting light.

In the decade following the publication of Hertz's results a number of different workers—Righi, Klemencic, A. D. Cole, Lebedew and others —extended Hertz's work by using what we now call microwaves (**9**).

With extremely short waves a few centimeters in length, generated at very low power, they were able to perform quasi-optical experiments which had not been possible with Hertz's somewhat longer waves (10). For example, Peter Lebedew in Moscow succeeded in demonstrating, by means of 6-mm. waves, that radio waves could be doubly refracted by crystals (11).

While the attention of physicists was quite generally focussed on the microwaves and their optical properties, interest was soon drawn to the possibility of wireless telegraphy by means of the new Hertzian waves. In his first successful transmissions on Salisbury Plain in 1896, Marconi used parabolic reflectors behind both his transmitter and receiver, and established communication at a distance of one and three-quarters miles using waves a meter in length.

Almost at once, however, he turned to much longer waves, principally because it was much easier to get high transmitter power at these wavelengths; also because they proved much better adapted to long range communication than shorter waves which are strongly attenuated over land or water. Still later, in 1901, Marconi's extraordinary success in bridging the Atlantic with radio waves revealed that certain of the lower frequencies, at least, were not limited in range by the earth's curvature.

THE RETURN TO SHORTER WAVES

During the Frst World War there was a sudden revival of interest in directed transmission, and hence in shorter wavelengths, because of the military possibilities of secret point-to-point beam communication. These possibilities were explored in 1917 by the French and by the Signal Corps Research Laboratory of the AEF in Paris; and a few years later by the United States Navy. The only work published in detail was that of Marconi and C. S. Franklin in 1916–17 (12). These men worked in England and in Italy, in collaboration with the Italian government, and got successful transmission up to 20 miles using waves of 2–5 meters. At the end of the war interest again shifted to longer waves, although the Marconi company continued to explore the possibilities of directed radiation.

Early experiments all served to confirm the opinion that short waves —in practice any wavelength much below 200 meters—were worthless for long-range communication (13). In consequence, the amateurs were relegated to the supposedly unprofitable region of 200 meters and below. Credit for discovering the remarkable properties of this region belongs to the amateurs who made up the membership of the American Relay League (14), the French *Société des amis de la T.S.F.* (15), and other associations.

In November, 1923, the tight little world of radio enthusiasts was startled to hear that a French amateur from Nice, Léon Deloy, had

established a two-way connection across the Atlantic with two American amateurs, F. H. Schnell and J. L. Reinartz (16). This extraordinary achievement advertised the peculiar advantages of "short waves" for long distance communication. In that year and the next, forcing their equipment to the limit, and operating vacuum tubes often at ten times their rating, the amateurs of America, Europe, Asia and Australia linked up the world by radio.

Industry followed closely on their heels. The Radio Corporation of America and the Telefunken Company's station at Nauen soon established commercial short-wave transoceanic links; in 1923 the Westinghouse station KDKA began broadcasting on 100 meters to Europe and America; in July, 1924, the Marconi company signed a contract with the British government and the Dominions to build four short-wave stations to connect the metropolis with India and the Dominions.

The amateur successes were also responsible for a redoubled activity on the part of the U. S. Navy whose radio engineers had just moved into the recently-completed Naval Research Laboratory at Bellevue, D. C. Under the leadership of A. Hoyt Taylor, Superintendent of the Radio Division at NRL, a program was launched, in close cooperation with the American Radio Relay League, to explore the behavior of short waves in long-distance communication (17).

The immediate effect of this research was to raise the question of why these short waves gave such unexpectedly long range. This work, which served to open up the field of ionospheric research, will be mentioned below. The second consequence was to inspire everyone, especially the amateurs, with the desire to see whether the favorable properties of the short waves might not be accentuated as one went farther down the spectrum. The exploration of shorter waves, touched off in 1923, continued unabated during the next decade. By the end of 1925 it had been demonstrated that long-distance daytime communication was possible on 20 meters.

It was some time before industrial concerns took the lead away from the amateurs, who in July, 1924, were authorized to use four new bands: 78–80, 40–43, 20–22 and 4–5 meters (18). Those who tackled the lowest wavelength discovered that it did not enjoy the long-distance virtuosity of 20 meters. Moreover, the techniques of producing and receiving 5-meter waves required departures from customary methods, whereas conventional tubes served adequately for 20 meters. In 1927 the amateurs were allotted a new band at 10–10.7 meters. Soon after its allocation a few dozen amateurs tried it out, hoping it might duplicate the performance of 20-meter waves. The results were disappointing—mainly it turned out later, because of the sunspot cycle which in 1929 caused serious magnetic disturbances.

After 1930, industrial laboratories put an increasing emphasis upon the study of short and ultra-short waves. Important papers appeared

from industrial laboratories on directive antennas and on propagation characteristics of ultra-high frequency waves (19).

At about this time widespread attempts were made to study transmission on waves 5 meters or shorter. In 1930 the Germans reported the results of successful communication experiments on 3 meters, obtaining a range of 180 miles with a 1.5 kw. transmitter (20) and demonstrating that on these frequencies line-of-sight limited the range. In France, tests showed the feasibility of ultra-high-frequency telephonic communication on 5 meters between Nice and the Col de Teghine in Corsica. An English experimenter reported similar work on 2 meters. Between 1930 and 1935 the commerical concerns made careful propagation measurements on these wavelengths—with application to television of these frequencies clearly in mind—and confirmed the German findngs as to the limited range. RCA made plans with the Mutual Telephone Company of Honolulu to explore the possibilities of ultra-high frequency waves for the establishment of inter-island telephonic links, and decided it could be done on 7 meters. In 1931, in the summer issues of *QST*, the organ of the American Radio Relay League, a campaign was launched to interest amateurs once again in the problem of communication on 5 meters. In 1934 Hull established an experimental 56-megacyle link between Boston and West Hartford, publishing the results in 1935 as an important pioneer study of line-of-sight propagation (21).

TUBES AND TECHNIQUES FOR ULTRA-HIGH FREQUENCIES

Thus by the early 1930's, just as the need for radio detection equipment was becoming acute, the course of radio experimentation had reawakened an interest in ultra-high frequency radio waves. Electronic television at about this time provided an added incentive for the development of new techniques and devices.

It is hard to imagine radar without the cathode-ray oscilloscope. After an uneventful development dating from the last century, this indispensable instrument first appeared about 1930 as a compact, lightweight and, above all, reliable device. It was at once adopted in all branches of electronic research.

The cathode-ray oscilloscope was invented by Ferdinand Braun in 1897 (22). His was a very simple device with a flat-disc cathode, a wire anode fed in from the side, an annular diaphragm to limit the beam and a fluorescent screen of zinc sulfide. It contained air at low pressure, and closely resembled the tube used by J. J. Thomson in determining the value e/m for electrons. The Braun tube in its early form was used by Zenneck and his school in studying radio circuits and the transmission of radio waves. In 1905 an important improvement, not immediately effective, was made by Wehnelt when he suggested the use of a hot, lime-coated filament as the source of electrons. This ultimately made it possible to operate tubes at reasonable voltages. The use of

early tubes was restricted because the equipment was large and cumbersome, because it required such high voltages, and because of the difficulty of maintaining a reliable vacuum. By about 1930, industrial research laboratories in the principal countries had produced compact hot-cathode tubes, operating on low voltages, that were sensitive, reliable and moderate in price. In the next few years the cathode-ray oscilloscope supplanted the mechanical oscillograph as a laboratory instrument, and found applications in a new and important field where even the best mechanical oscillographs were inapplicable: the study of wave-shapes in the higher radio frequencies.

The influence of television upon the development of radar was not as direct as the superficial similarity of the two types of equipment would indicate. Television and radar were parallel developments, and because of the secrecy which surrounded radar they did not often interpenetrate. The earliest experimental television was accomplished in 1925–6, with mechanical scanning devices like the Nipkow disk or the later perforated drum, used for both pickup and viewing (**23**). Two important developments in its history took place in the early 1930's: (1) the abandonment of mechanical systems with the recognition that the future of television lay in using the cathode-ray tube both for pick-up and for viewing, as A. A. Campbell-Swinton had suggested as early as 1908, and (2) the decision to adopt ultra-high frequencies for television transmissions, despite propagation difficulties. Since television transmitters must radiate an extremely broad bandwidth of frequencies in order to transmit pictures of high definition, it was clearly advantageous to use ultra-high frequencies.

After 1930, television provided the chief commercial incentive for opening up the ultra-high frequency region. Tube and circuit development, propagation studies, and above all systematic and continued development of the cathode-ray oscilloscope were the chief dividends reaped by radar from television research during the first years. In the decade before the war, important work went forward in improving the fluorescent screen, studying the properties of phosphors, and improving the mechanical structure of cathode-ray tubes. To this should of course be added accumulated experience in the design of scanning and synchronizing circuits. Much of this influence was only felt on the eve of the war and during the war when television engineers, like ionospheric investigators, were recruited for radar work.

A large literature grew up describing the special techniques for using ultra-high frequency waves, especially the design of transmitting and receiving tubes. By about 1930 it became evident that the limit was being approached above which conventional tubes could not be dragooned into operating (**24**).

There are inherent limitations in conventional tubes that become increasingly critical as frequency is raised. Above 10 megacycles per

second (Mc./sec.) the efficiency of tubes begins to show a sharp decline.

In the 1930's the chief electronic concerns were producing tubes specially designed for ultra-high frequencies. The Eitel-McCullough Co. in California brought out rugged transmitter tubes for the use of amateurs. And by 1934 RCA acorn tubes and Western Electric "door-knob" tubes were on the market (25). These were tubes of small dimensions with low transit time, short leads with low inductance and capacitance and very small inter-electrode capacity.

These improvements disposed of the major obstacles to pushing down into the region between 3 meters and 1 meter. Radio communication at these wavelengths, subject of course to certain limitations prescribed by the characteristics of the waves themselves, was thus clearly within reach. But to penetrate the realm of the microwaves or centimeter waves was to require a whole new art. The earliest radar, however, did not presuppose proficiency in microwaves, and radio detection devices between 10 meters and $1\frac{1}{2}$ meters were strongly influenced by the developments just enumerated.

OBSERVATIONS ON REFLECTION

With the shift to very long waves, the phenomenon of reflection was no longer readily observed and did not come into prominence until work was resumed on ultra-high frequencies during and just after World War I (26). Marconi spoke of it in his New York address in 1922, and A. Hoyt Taylor referred to it in an article published in *QST*, early in 1924 (27).

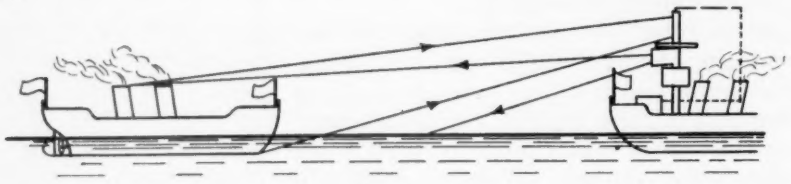

FIG. 1.

After publication in *QST* the phenomenon must have been well known to radio enthusiasts (28). Numerous references appeared in print after 1930 (29), including a report by engineers of the Bell Telephone Laboratories which has considerable historic significance as the first published account of the detection of radio energy reflected or reradiated from an aircraft. The authors spoke of the general phenomena as being "well known," but emphasized their discovery that the effect "extends to unsuspected distances at times," notably in the case of reflections from aircraft which they studied with care (30). Identical but unpublished observations had an important influence upon radio detection at the U. S. Naval Research Laboratory and in Great Britain.

It was only a step from this to the idea that such reflections could be used to detect objects in fog, darkness or limited visibility. The earliest proposal of this sort that has come to light is the patent granted in several countries to an engineer of Dusseldorf, Christian Hulsmeyer, in 1904 for a collision prevention device (see Fig. 1). This consisted of a spark-gap transmitter sending out a beam of very short radio waves focussed by means of a hemi-cylindrical reflector located on the ship's mast. The echo is picked up by a receiving antenna also provided with a reflector. The cw. signal is detected by the coherer of a shielded receiver. The device, mounted as in a compass box to stabilize it, was rotated to determine directions (31).

No other proposal seems to have been made until after the First World War. On June 20, 1922, Marconi was guest of honor at a joint meeting in New York of the Institute of Electrical Engineers and the Institute of Radio Engineers. The occasion was the presentation to Marconi of the I.R.E. Medal of Honor in recognition of his work in wireless telegraphy. In a long address he described his communication experiments using waves a few meters in length and directional transmitters, called attention to the phenomenon of reflected radio waves, and suggested their use for detection (32):

> It seems to me that it should be possible to design apparatus by means of which a ship could radiate or project a divergent beam of these rays in any desired direction, which rays, if coming across a metallic object, such as another steamer or ship, would be reflected back to a receiver screened from the local transmitter on the sending ship, and thereby immediately reveal the presence and bearing of the other ship in fog or thick weather.

The earliest experimental confirmation of these speculations took place in the autumn of 1922 at the Naval Aircraft Radio Laboratory—later to become the Naval Research Laboratory—at Anacostia, D. C. The observers were A. Hoyt Taylor and Leo Clifford Young. About the middle of September, 1922, they began an informal investigation of 5-meter waves.

Their receiver was installed in an automobile and the first experiments were made on the grounds of the Naval Air Station, the transmitter being set up near the door of their laboratory, only a few feet above the ground. Almost at once interference effects were noticeable. As the car drove away from the transmitter and passed certain steel buildings from which some of the radiation was reflected a very sharp series of maxima and minima were observed. The sound was now intensified and now weakened as the car moved away from the transmitter, clearly resulting from the simultaneous reception of energy coming directly from the transmitter and reflected from the buildings.

Such things as a passing automobile, a screen door, or a network of wires, like the backstop of a tennis court, caused marked shielding

effects. Experiments on the transmission of the energy over water were tried by driving the car with the receiver to Haines' Point across the Potomac. The same interference effects were also noticed, and it was discovered that they came from clumps of willow trees near the receiver. While these experiments were in progress and unobstructed signals were being received from the other side, the steamer *Dorchester*, a wooden vessel of no great size, passed down the channel. Fifty feet before the bow of the steamer crossed the line of vision between transmitter and receiver, the signals jumped to nearly twice the previous intensity. When the steamer actually passed across this line they dropped to half the normal value. Again when the stern of the vessel had passed fifty feet farther downstream, the signals rose to normal intensity, then to about twice normal, and then dropped back again.

September 27, a memorandum drawn up by Taylor was transmitted by the Commanding Officer of the Station to the Bureau of Engineering, describing the experiments and urging the possibility of directive communication on this wavelength. It also referred to the possibility of using highly directive beams for landing aircraft at night or through overcast. The report (33) urged also that "possibly an arrangement could be worked out whereby destroyers located on a line a number of miles apart could be immediately aware of the passage of an enemy vessel between any two destroyers of the line, irrespective of fog, darkness or smoke screen." The memorandum requested Bureau approval for continuance of the 5-meter work, but no encouragement was forthcoming and it was dropped. This can be considered the earliest proposal for the use of radio detection as a military device.

It was not until 1930 that the subject of radio detection raised its head once more, this time in print. In the July issue of the *Proceedings* of the Institute of Radio Engineers an English contributor wrote (34):

> It has already been suggested that icebergs, etc. might be detected by short-wave radio and it is possible, for instance, that this would be done by projecting a short-wave beam and observing whether any energy were reflected back to the source. A similar method might be employed for estimating the height of aircraft above ground.

PULSE RANGING AND THE PULSE-ECHO PRINCIPLE

The earliest efforts to measure distance by the careful timing of pulses of energy were associated with the use of sound for depth measurement, a technique that appears to have been first suggested by the French physicist Arago in 1807. Early attempts were made to measure the ocean depths by detonating from the surface a heavy charge of gunpowder on the bottom of the sea and determining the interval before the sound was heard (35). This method was successfully used by Behn in measuring the depths of Lake Ploen in 1912. A more sophisticated ap-

plication of this idea was the British Admiralty depth-sounding machine where a steel hammer striking a steel plate in the bottom of a ship, sent out a highly damped compressional wave, picked up after reflection by a hydrophone. In the "fathometer" of Fessenden, the transmitter was an oscillator sending out a short pulse on a frequency of a few thousand cycles per second. In the years following the First World War the principal maritime countries all announced successful trials of echo-sounding equipment, sonic or supersonic.

The *Titanic* disaster had led to Lewis Richardson's proposal that high-frequency supersonic beams could be used for depth measurement and obstacle detection at sea. Such devices became practical after the Langevin and Chilowsky system had produced supersonic beams at 30,000–40,000 cycles per second. Their use of the piezo-electric crystal found an important application in a supersonic pulse device for submarine detection. In World War I development work on these devices went forward in America in the New York and San Pedro groups under the National Research Council. Although it was developed too late to be of use, a supersonic device that could detect submarines half a mile or more away had been built by the end of the war. The Navy continued work on this device. With many improvements, it became the modern *sonar* equipment of World War II. By a simple extension of the basic principle, supersonic altimeters for aircraft were soon thought of.

Between 1920 and 1940 numerous patents were granted and many articles were published on possible methods of using radio for distance measurement. This work had three purposes: (1) point-to-point surveying and geodetic measurements by radio; (2) altimeters for aircraft; (3) exploration of the Kennelly-Heaviside layer. In each the use of pulses was at some time or other proposed.

About 1930 research on finding the distance between two fixed points by radio was undertaken in the Soviet Union. Several different methods were considered including the use of pulses (**36**). With their final method, a phase displacement method, the Russians in 1934 made a series of 900 measurements in the North Caucasus, shooting between mountain peaks and from peaks to valleys.

Many proposals were made during the first decade after World War I for using reflected radio waves for aircraft altimeters. The patent literature before 1930 yields a wide variety of suggestions for utilizing the reflection of waves directed downward from a plane (**37**). Of greatest interest, and generally of greatest simplicity, are devices depending on simple measurement of the time required for radiation to travel from a transmitter to a reflecting surface and back. Here several paper patents closely resemble or actually anticipate the pulse method.

The earliest disclosure of this sort appears to be that of an Austrian, Henrich Löwy, in a patent application filed in July 1923 (**38**). The patentee describes his proposal as the electric counterpart of Fizeau's

method for determining the velocity of light. In Löwy's scheme an electronic switch is used to key a transmitter and synchronously to block and unblock a receiver, so that while the transmitter is sending out a short pulse, the receiver is off; and when the transmitter is silent, the receiver is unblocked and ready to receive the echo signal. The blocking and unblocking are produced by an imposed sinusoidal modulation the frequency of which can be varied, but which is of the order of broadcast frequencies. In radar parlance, this device has a pulse width roughly equal to the resting time. When the modulation fre-

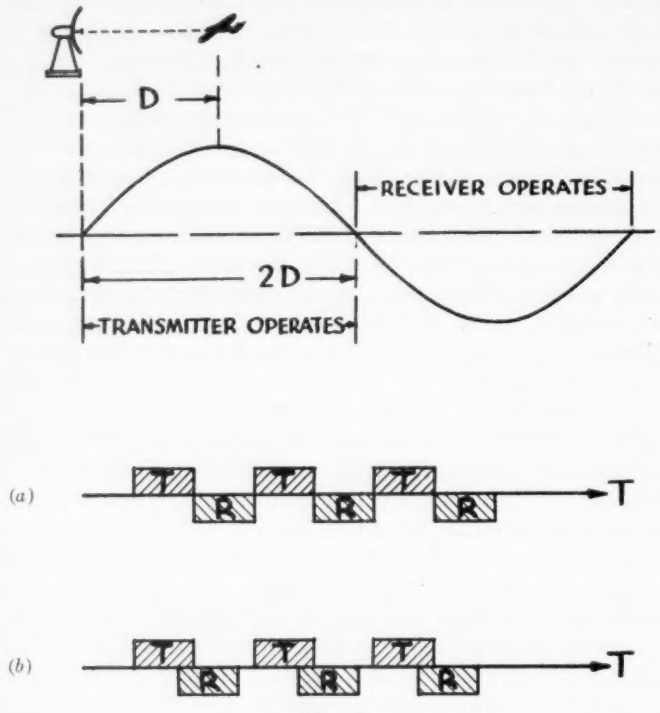

FIG. 2.

quency is chosen so that the half wavelength corresponds to twice the distance to the reflecting object, then the situation corresponds to Fig. 2. The reception of the reflected energy begins immediately at the end of the transmission, as in Fig. 2(*a*). When the distance is less than a quarter of the modulation wavelength, the situation corresponds to Fig. 2(*b*) and the returning echo reaches the receiver while the transmitter is still operating. Conversely, when the distance is greater, the received echo returns sometime after the cessation of the transmitter, and continues to arrive after the receiver has been shut off. The output of the receiver is fed to a meter or a gas discharge tube as indicator. Since the

maximum signal is received only when the modulation frequency is properly chosen, varying this frequency within convenient limits makes it possible to determine altitude. The essential difference between this pulse device and radar should be evident. In May, 1930, two interesting patent applications were filed by Ezekiel Wolf and by Robert W. Hart and assigned to the Submarine Signal Company of Boston, Mass. (39). Both were for schemes of distance measurement by reflected radio waves, principally for aircraft altimeters, and both provided for the use of pulses of energy. (Wolf's patent applied to continuous waves as well as pulses.)

Hart's patent is for a pulse device corresponding in all essentials to that devised a few years before by Breit and Tuve, which we shall discuss below. Though the language tends to obscure the fact—for example, Hart speaks of "trains of vibration" instead of pulses, and of these trains as being produced by "overmodulation"—his patent is clearly for a pulse-echo altimeter. The pulses from the transmitter are reflected from the ground or other object and the echo is picked up by the receiver and is put on a cathode-ray tube with a circular trace. The cathode-ray trace makes one complete rotation for each cycle of the modulation frequency. The trace can be easily calibrated and the position of the "momentary serration" read as distance to the ground or other target. It differs in no important respect from a nondirectional pulse altimeter.

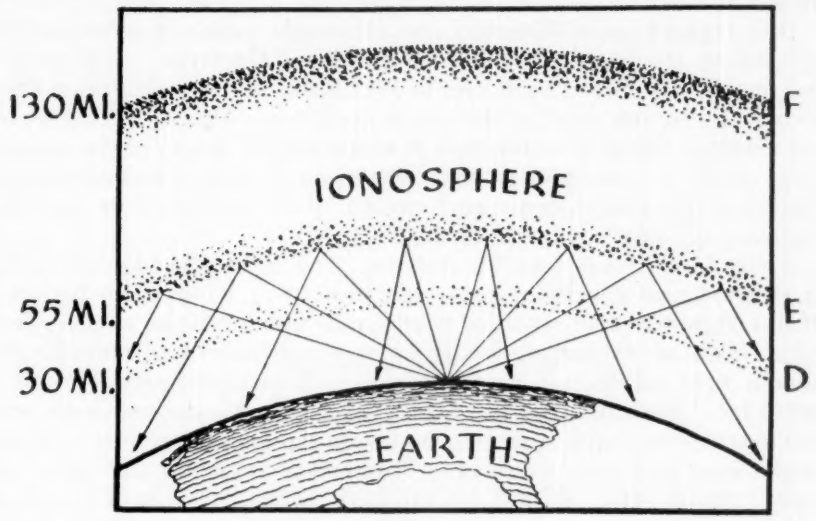

FIG. 3.

But the pulse-echo method became best known throughout the scientific world through its use in studying the properties of the *iono-*

sphere. This is the ionized region of the upper atmosphere of which the lower layer is called the Kennelly-Heaviside layer after the two workers who independently inferred its existence (see Fig. 3).

Marconi's dramatic success, on December 12, 1901, in receiving signals transmitted across the Atlantic from Cornwall to Newfoundland, led to speculations as to the mechanism which made this possible. Arthur Edwin Kennelly in America and Oliver Heaviside in England independently suggested within a short space of time that these waves were propagated around the curvature of the earth with the assistance of a conducting surface in the upper atmosphere; the waves moving, in Kennelly's words (40), "horizontally outwards in a 50-mile layer between the electrically-reflecting surface of the ocean beneath, and an electrically-reflecting surface, or successive series of surfaces, in the rarified air above."

The suggestions of Kennelly and Heaviside were not accompanied by a serious attempt to explain the formation of the layer. A quantitative theory was offered by Eccles in 1912 based on the assumption that the waves were refracted by the ions in the layer. Although evidence of various kinds began to accumulate in support of the theory of the ionized layer, systematic verification was delayed until interest was once more kindled by the long-distance performance of 100-meter waves, and the discovery of "skip" distance during 1924 (41).

Early in the year John L. Reinartz, one of the best-known radio amateurs, began a series of long-range tests using wavelengths between 20 and 60 meters, and observed that signals which became weaker after sunset (instead of stronger, as with waves about 100 meters) surprisingly enough were audible at great distances. Thus there seemed to be a "dead belt" lying between the region covered by the ground wave and that reached by a sky wave returned from the ionosphere (42). These results were confirmed by A. Hoyt Taylor who named it the "skip" or "miss" region (43). With E. O. Hulburt, Superintendent of the Heat and Light Division of the Naval Research Laboratory, he offered a modified Eccles-Larmor theory to account for the new phenomenon, making it clear that these very short waves are propagated for short distances in the conventional manner (44). But in addition to the ground ray, which quite rapidly became attenuated, sky waves reached great distances by being sharply refracted by the ionosphere. In regions where both the ground wave and a sky wave are received violent fading is encountered, due to interference effects between the two waves.

The stage was now set for the experimental proof of the layer's existence. In 1925 Smith-Rose and Barfield in Great Britain applied direction-finding methods on the assumption that there should be a detectable difference in angle between the electric (and magnetic) vectors of the ground and sky waves (45). Their results were inconclusive, though in 1926 they confirmed observations reported in the interim by

two other English workers, E. V. Appleton and M. A. F. Barnett (46).

Appleton and Barnett were the first to use reflected radio waves for the radio-location of a distant surface and the determination of its range (47). Their method was an adaptation of techniques used in Appleton's earlier work on radio signal fading and involved intensity measurements instead of direction-finding methods. They believed that for wavelengths from 300–500 meters there was a point about 100 miles from a given transmitter, where the strength of the ground wave and the sky wave would be sufficiently comparable for strong interference effects to be produced. By varying uniformly the frequency of the transmitter by a known small amount in a brief time—what we now refer to as frequency modulation—it was thought possible to observe directly the interference effects at that point and to determine the distance of the

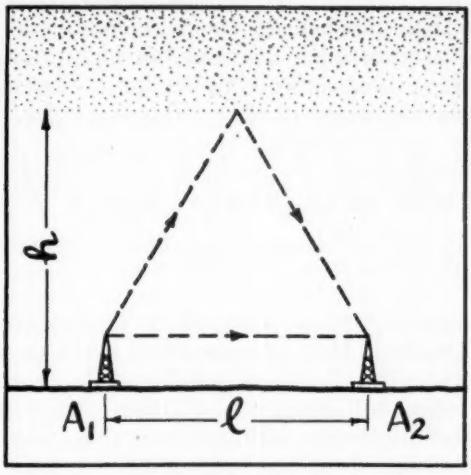

FIG. 4.

reflecting layer. In experiments performed on the nights of December 11, 1924 and February 17, 1925, using the British Broadcasting Company's transmitter at Bournemouth, Appleton and Barnett obtained interference effects which were the first direct evidence of the existence of the reflecting layer. They estimated it to be at a height of 80–90 kilometers.

Two American investigators, Merle A. Tuve and Gregory Breit, who undertook an ionospheric investigation without being aware of the similar work going forward in England, were the first to use the simpler radio technique of transmitting discontinuous pulses and measuring the time for the returning echo to reach a nearby receiver.

Their method grew out of one tried at the University of Minnesota by W. F. G. Swann and J. G. Frayne, and, like Löwy's altimeter, used

electrical shuttering to block the receiver when the transmitter was sending and unblock it for receiving the echo (48). Breit and Tuve proposed sending out pulses of such a length and repetition rate that at a receiving station some distance away a pulse from the ground wave and the sky wave could be distinguished. From the delay separating the two received pulses, and the known distance between the two sta-

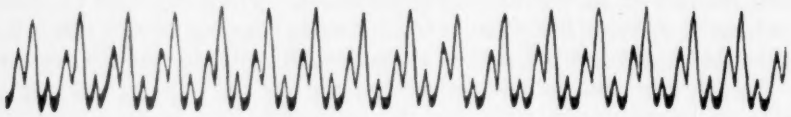

FIG. 5. Oscillogram from the earliest experiments of Breit and Tuve, 3:15 P.M., July 28, 1925, showing ground wave, and double reflections from the Kennelly-Heaviside Layer, using 71 meter transmitter at the Naval Research Laboratory. Pulses produced a-c. modulation on transmitter plate.

tions, the height of the ionosphere could be estimated. The arrangements described in the first paper are shown in Fig. 4 where A_1 is the transmitting station, A_2 the receiving station, l the distance between the stations, and h, the height of the conducting layer.

FIG. 6. Early Breit and Tuve oscillogram (September 25, 1925, 3:30 P.M.) showing single reflection from ionosphere and effects of slight fluctuations of the layer.

The general suggestion of studying the reflecting layer by radio was due to Breit. Early in 1924 he proposed to Tuve, then an instructor in physics at Johns Hopkins, that they collaborate in an attempt to transmit radio waves from the Carnegie Institution's Terrestrial Magnetism Laboratory in Washington to Baltimore by reflecting them from the ionosphere. It was Breit's original plan to direct short waves skyward

FIG. 7. Early oscillogram obtained at Carnegie Institution (October 7, 1928) using multivibrator circuit for producing pulses. Note the long interval between successive ground waves (*G*).

by means of a large parabolic reflector of wire netting, about a 100 feet in diameter, and have the sky wave received by Tuve at Johns Hopkins.

Since the proposal of having the Terrestrial Magnetism Laboratory embark on radio research was something of a novelty, the possibilities were discussed at a meeting, held in November 1924, at which leading radio experts in Washington, like Austin and A. Hoyt Taylor, were

called in for consultation. It was during a discussion immediately after this meeting during dinner at Breit's house that Tuve proposed that they try to use pulses in the manner previously suggested by Swann and Frayne at Minnesota. At the close of the spring term in 1929 at Johns Hopkins, Tuve came to Washington and joined his colleague in these historic experiments (49).

Arrangements were made with the Naval Research Laboratory to use NKF, with Westinghouse for the use of their pioneer station KDKA, and with RCA and the Bureau of Standards. The best results were obtained with the Navy's transmitter, because of the advantageous location of the two laboratories, and because the Navy transmitter was crystal-controlled and had high stability. Gebhard, Schrenk and Young at NRL assisted in modifying the transmitter for pulse work and in building a receiver. Experiments were begun on July 28, 1925, with the receiver located at the laboratory of the Department of Terrestrial Magnetism, 8 miles southeast of the Naval Research Laboratory. Pulses lasting about 1/1000 sec. were transmitted on 71.3 meters. Pulse modulation was produced by an a-c. frequency of about 500 cycles per second applied to the plate circuit of the transmitter. The received pulses, recorded by an oil-immersed General Electric oscillograph, were observed visually by means of a rotating mirror and also photographed. From the time delay, Breit and Tuve estimated the height of the ionosphere as between 50 and 130 miles (50). Additional observations showed that reflections could be obtained using waves of about 40 meters in length, but that no reflections were obtained with waves below 20 meters. The reflections apparently varied with the time of day and with the seasons, which seemed to prove that the echoes could not come from ground reflections.

This method of Breit and Tuve was widely adopted as the simplest and most direct. A world-wide network of investigators was soon busy exploring the upper atmosphere by radio and building up increasingly detailed knowledge of the fine structure of the ionosphere, which was found to consist, not of a single region of ionization as was thought at first, but of two or three layers, depending upon whether it is day or night (51). In 1928 Tuve and Dahl described an improvement suggested somewhat earlier by G. Breit for pulsing the transmitter by a better method than the 500-cycle modulating e.m.f. which had produced pulses very close together, separated by an interval scarcely greater than the pulse length, and had led to difficulty in distinguishing multiple reflections, and to poor resolution for small distances (52). The new method involved the multivibrator circuit of Abraham and Block, later to play an important part in both television and radar, as a basic pulse-forming circuit. With this it was possible to get shorter pulses (one or two ten-thousandths of a second) separated by relatively long intervals (a few hundredths of a second).

The technique was improved also by substituting a cathode-ray tube for the mechanical oscillograph. In 1930 Georg Goubau reported the use of a cathode-ray tube with a circular or elliptical trace, synchronized with the pulse cycle (53). In 1931 Appleton reported simultaneously the adoption of the cathode-ray tube with linear trace as an indicator for ionosphere experiments and the use of a self-pulsing or "squegging" transmitter (54). Except for the frequencies employed, the chosen pulse lengths and intervals between the pulses, and the absence of directive antennas, the apparatus used for ionospheric work by the early 1930's bore considerable resemblance to a radar system. It was the ionospheric investigations, moreover, which spread abroad knowledge of the pulse-echo principle, and reduced the art of radio-location by pulses to the level of daily experience.

Radar was developed by men who were familiar with the ionospheric work. It was a relatively straightforward adaptation for military purposes of a widely-known scientific technique, which explains why this adaptation—the development of radar—took place simultaneously in several different countries.

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(34) W. J. BROWN, "Ultra-short waves for limited range communication," *Proceedings of the I.R.E.*, Vol. 18, p. 1142 (1930).

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(38) LÖWY, U. S. Patent 1,585,591, filed July 17, 1923.

(39) WOLF, U. S. Patent 1,924,174; Hart, U. S. Patent 1,924,156, both filed May 19, 1930.

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THE SPECIFIC IONIZATION CURVES FOR HYDROGEN AND DEUTERIUM

BY

T. N. HATFIELD,[1] A. E. LOCKENVITZ,[1] AND J. M. YOUNG[1]

ABSTRACT

The specific ionization (Bragg) curves are obtained for hydrogen and deuterium, with alpha particles from polonium as the ionizing source. The curves are found to coincide quite closely and the stopping powers as determined from the curves are the same for both gases.

The relationship between the stopping power of an element for alpha particles and the atomic weight or atomic number of the element has been under study for some time. Bragg and Kleeman (1)[2] found an empirical relationship that the stopping power is proportional to the square root of the atomic weight, the relationship holding quite well for elements which had been studied at that time. Bohr (2), Bethe (3) and Bloch (4) have, from theoretical considerations, derived expressions for the stopping power of a substance for high velocity particles. The basis for the theoretical work is the assumption that the particle loses most of its energy in collisions with electrons rather than with nuclei, so that the molecular stopping power comes out as a function of the atomic number instead of the atomic weight.

The relation given by Bloch for the relative molecular stopping power of an element for alpha particles may be written in the form:

$$S_{12} = \frac{Z_1(K - \log Z_1)}{Z_2(K - \log Z_2)}, \tag{1}$$

where S_{12} is the atomic stopping power of element of atomic number Z_1 relative to the atomic stopping power of element of atomic number Z_2 and K is a numerical constant for all elements. The value of K may be determined from the stopping power of one element relative to another.

Although the theoretical considerations give the stopping power as a function of atomic number, many authors or articles and text books continue to give the stopping power as a function of atomic weight. Most of these are based on the Bragg relation of stopping power proportional to the square root of the atomic weight, a relation which holds only in a limited range. E. Segre (5) states that the mass stopping power (stopping power divided by the density) is inversely proportional to the square root of the atomic weight. This will result from applying the Bragg

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

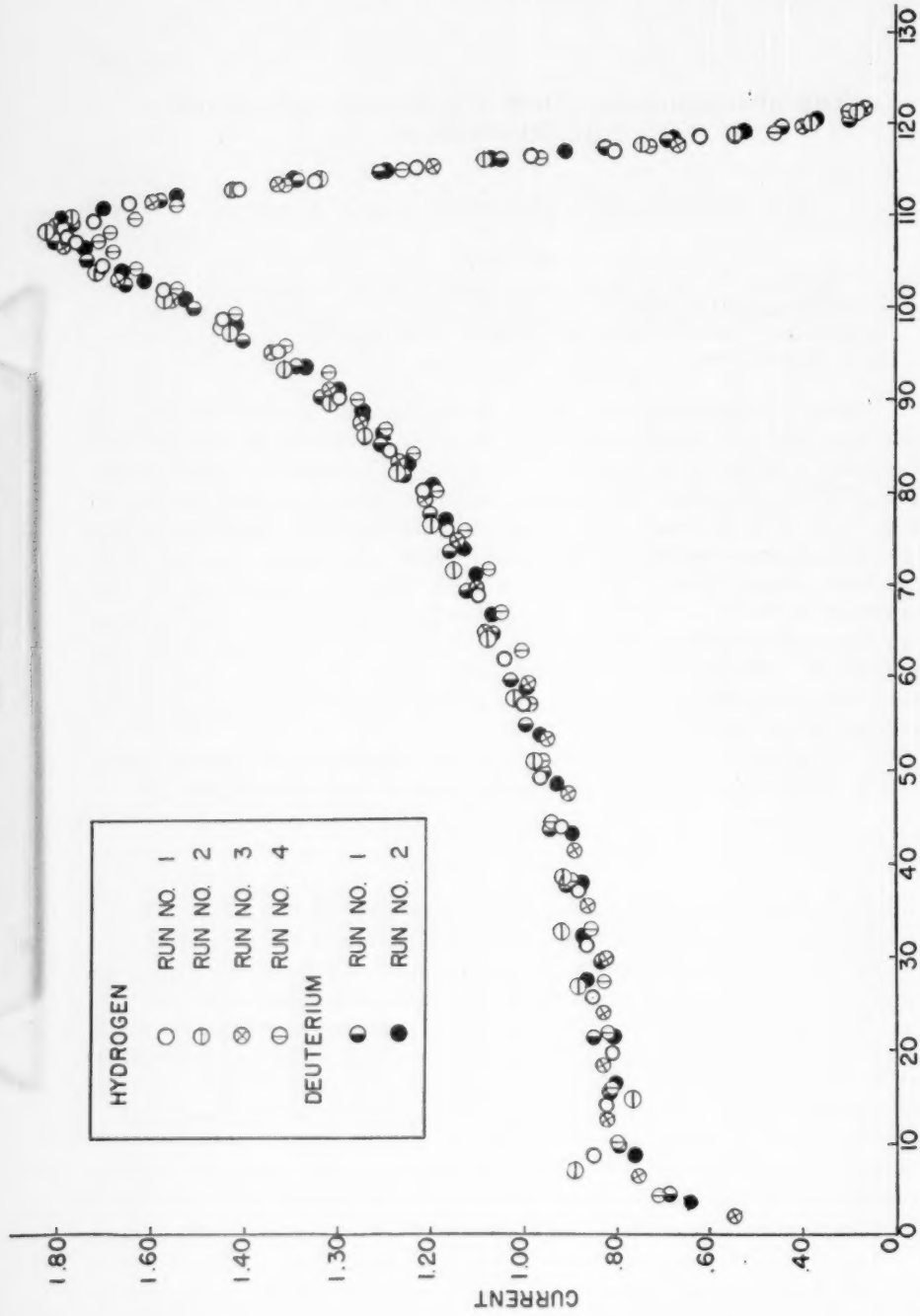

FIG. 1. Composite specific ionization curve for hydrogen and deuterium.

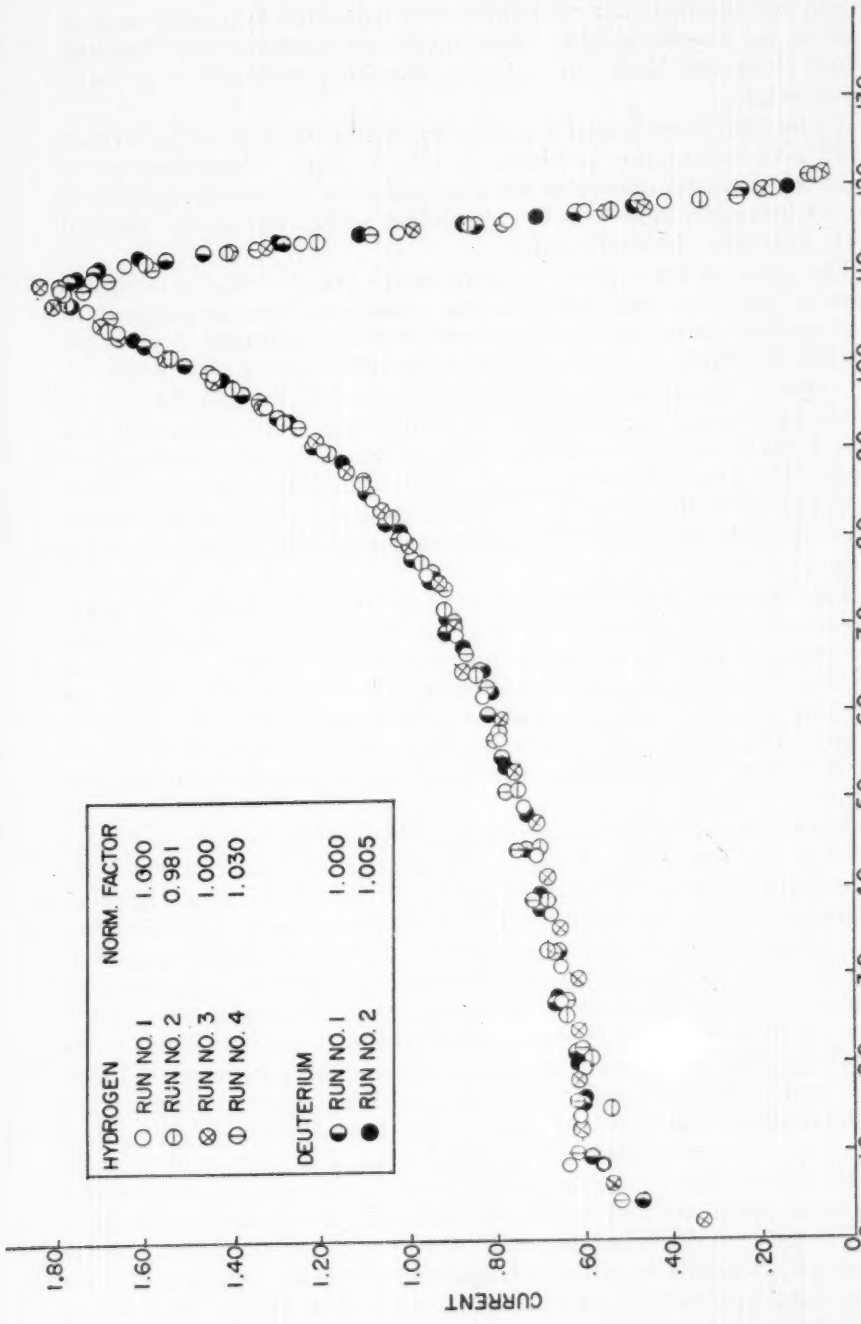

FIG. 2. Normalized composite specific ionization curve for hydrogen and deuterium.

relation and the fact that the density of a substance is roughly proportional to the atomic weight. Richtmyer and Kennard (6), Glasstone (7) and Hoag and Korff (8) also give stopping power as a function of atomic weight.

A study of the molecular stopping power of hydrogen and deuterium should give information pertinent to this problem. According to the Bragg relation, the molecular stopping power of deuterium relative to that for hydrogen should be $\sqrt{2}$. According to Eq. 1 from the work by Bloch, this ratio should be unity.

The authors have made measurements on the relative stopping power of hydrogen and deuterium for alpha particles from polonium. The complete ionization curves were obtained to determine, in addition, whether the difference in the nucleus would affect any part of the ionization curve. The specific ionization apparatus of Colby and Hatfield (9) is used in making the ionization measurements. This apparatus employs a fixed source-to-electrode distance, and ionization currents are measured as a function of the gas pressure. Hatfield, Colby, and Lockenvitz (10) have discussed the validity of using the extrapolated pressure range with this apparatus in determining the relative stopping power of gases.

The hydrogen and deuterium were purified by passing the gases through activated charcoal at liquid air temperature. Measurements were made on two samples of hydrogen, then two samples of deuterium and then two other samples of hydrogen. An attempt was made to use the same arbitrary units of current in each measurement by correcting for any change in the gain of the amplifier and by correcting for the decay in source strength during the five days required for the measurements.

The results of the six measurements are given in the curves of Fig. 1 and Fig. 2. The curve of Fig. 1 was plotted from data in which the ionization current had been corrected to 15° C. and 760-mm. Hg pressure and the corrections mentioned above had been made. In this figure the curves formed by the points plotted from each run deviate only slightly from one another, indicating that the current corrections for change of amplification and source decay are quite satisfactory. However, small normalizing factors applied to the currents in some of the runs bring them into even closer agreement, and this is indicated in Fig. 2.

Schmitt, Hatfield, and Lockenvitz (11) have reported the stopping power of deuterium to be the same as that for hydrogen. It is evident from Fig. 2 that not only is the stopping power independent of the mass of the nucleus, but the entire specific ionization curves are the same for hydrogen and deuterium, within the limits of the experimental error.

The value of the stopping power of hydrogen and deuterium relative to air as calculated from the extrapolated pressure range is 0.223, the same value as reported for hydrogen by Hatfield, Colby, and Lockenvitz (10).

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Telephone Service for the Pentagon Building (*Bell Laboratories Record*, Vol. XXVIII, No. 8).—What will be the world's largest private branch system is coming into being in Washington for the Department of Defense. A step-by-step system, it was manufactured by Western Electric and furnished to the Government by the Chesapeake and Potomac Telephone Company. It will have 18,500 lines and 33,000 telephones and will handle upwards of 250,000 calls a day. By calling Liberty 5-6700, any telephone in the Pentagon can be reached; and by October when Navy telephones are transferred, the same number will reach them.

Large enough to serve a city of 200,000 people, the new system could be nearly doubled in size in an emergency. Incoming calls to the Defense Department will be handled at the Pentagon switchboard being used eight hours per day and five days each week.

Magnetic Amplifier (*General Electric Review*, Vol. 53, No. 7).—With magnetic amplifiers assuming a role of increasing importance in control applications a demonstration unit has been designed for educational purposes in schools and industry. The unit is so arranged that all basic single-phase self-saturating circuits can be studied. Because the characteristics of amplifiers connected for d-c. and a-c. output and control differ, it is important to be able to study all of these circuits and their combinations. Included with each unit are wiring diagrams of control circuits, as well as laboratory experiments arranged to help the student determine static and dynamic characteristics, internal impedance and optimum load, and types of control current. The unit is manufactured by Vickers Electric Division.

New Battery Expected to Find Wide Application.—A non-renewable, air-depolarized add-water type primary battery, the "Carbonaire," recently introduced by Thomas A. Edison, Inc., is expected to find application in many fields.

First widespread use will be in converting oil-burning railroad switch-lamps to electrical operation at a saving to the railroads of as much as $25 per lamp per year. This saving is possible because of the elimination of weekly or twice-weekly servicing. The new Carbonaire battery will supply enough electricity to keep a switch-lamp going for ten months to a year with only occasional inspections.

The present size Carbonaire, and others now being planned, will be suitable for many applications such as fire and burglar alarm systems, annunciator systems, elevator signals, laboratory apparatus, aids to navigation, telephone and telegraph installations, time clock systems, communication systems and other services that require a reliable, independent source of low voltage d-c. energy.

The 1000 amp.-hr. model now in production contains two series cells sealed in a single, molded, hard rubber case. Dimensions are $8\frac{1}{4}$ by $7\frac{1}{4}$ by $9\frac{1}{2}$ in. Like other primary batteries, the Carbonaire is not chargeable and is replaced when expended.

STEADY FORCED VIBRATION OF A NON-CONSERVATIVE SYSTEM WITH VARIABLE MASS; A PUMPING SYSTEM

BY

ROBERT S. AYRE,[1] LYDIK S. JACOBSEN,[1] AND ARIS PHILLIPS [1]

ABSTRACT

There is an interesting class of vibratory systems in which the mass is a function of the velocity of the system. This paper treats, both theoretically and experimentally, the steady forced vibration of a single-degree-of-freedom system, with stepwise variation in mass, and by which external work is done. The deep-well reciprocating pump has been used as an example of such a system although in doing so it is necessary to consider the pump a highly idealized, undamped system with lumped parameters. The main purpose has been to study the effect of variation in mass. The theory considers two linear systems, one of mass M, the other of mass $M + m$, the boundary conditions of which must be made to agree. Four distinct "Classes of Motion" are shown; three of them involve an over-travel of the secondary mass, m. The experimental apparatus is mechanical.

NOMENCLATURE

k = equivalent spring constant of the piston rod.

M = equivalent mass of the piston rod and piston (the *primary* mass).

m = equivalent mass of the fluid column (the *secondary* mass).

w = weight of the *unbalanced portion* of the fluid column.

p = natural frequency for *connected* motion $= \sqrt{\dfrac{k}{M+m}}$.

q = natural frequency for *disconnected* motion $= \sqrt{\dfrac{k}{M}}$.

$\delta = \dfrac{w}{k}$ = static deflection of the piston caused by the unbalanced fluid.

t = time.

l = time interval of *connected* motion in one cycle (a negative time).

λ = time interval of *disconnected* motion in one cycle (a positive time).

x = displacement of the piston, also of the fluid column, even when the two differ.

S = amplitude of the forcing motion.

ω = forcing frequency.

ϕ = phase angle of the forcing motion with respect to the piston motion.

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$\mathfrak{M}$ = magnification factor; that is, ratio of the piston stroke to the forcing stroke $2 \cdot S$.

$$P = \frac{1}{1 - \frac{\omega^2}{p^2}}; \quad Q = \frac{1}{1 - \frac{\omega^2}{q^2}}; \quad R = \frac{1}{1 - \frac{p^2}{q^2}}.$$

slip = displacement of fluid column in one cycle minus the piston stroke, that is, the over-travel in one cycle due to the secondary inertia.

INTRODUCTION

The purpose is to obtain an understanding of the dynamics of a particular class of systems with variable mass. The deep-well reciprocating pump has been idealized to provide a convenient example of the

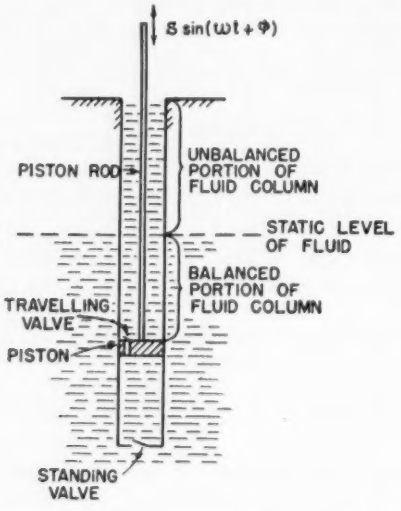

FIG. 1. Idealized pumping system. Note that normally the length of fluid column between piston and standing valve is negligible compared to length of column above piston.

class; the paper does not, however, attempt to arrive at operating formulae for such pumps. The emphasis is placed on the effects of stepwise changes in mass. Several papers which relate more specifically to the deep-well reciprocating pump have appeared in the last ten or fifteen years. From the applied mechanics point of view, those by Kemler[2] and by Langer and Lamberger[3] are perhaps the most interesting.

The pump (Fig. 1) is essentially a vibrating system which, during the

[2] EMORY KEMLER, "An Investigation of Experimental Methods of Determining Sucker-rod Loads," *Transactions A.I.M.E.*, Petroleum Div., Vol. 118, 1936, pp. 89–99.

[3] B. F. LANGER AND E. H. LAMBERGER, "Calcuation of Load and Stroke in Oil Well Pump Rods," *Journ. Applied Mechanics*, Vol. 10, No. 1, March, 1943, pp. A-1, A-12.

down stroke has only the rod and piston for its mass elements, while for the upstroke motion the mass of the fluid column must be considered as well. Moreover, a pump is a non-conservative system since work is done in lifting part of the fluid column. The practically important questions for damping, throttling effect of the valves, and distributed mass and elastic properties of the piston rod, pump casing and fluid column have been omitted.

It was found experimentally, and later analytically, that under some conditions the "fluid column" continued to move upward after the "piston" had started on the down stroke. This effect was particularly pronounced when the system was operating in the region of resonance, and also when the gravity term, that is, the unbalanced head on the piston, was small.

THEORY

Assumptions

(1) The system behaves as two single-degree-of-freedom systems, alternating with mass M, and with mass $M + m$. At the instants of transition from one system to the other displacement continuity must prevail. (2) The secondary mass m (fluid column) cannot have negative velocity. (3) Damping is negligible. (4) The forcing motion, $S \sin (\omega t + \phi)$, is independent of the pistion rod load, that is, there is sufficient power available.

Differential Equations of Motion

Two general conditions are possible:

(1) *Disconnected motion.* If the velocity of M is less than that of m, k and M constitute a single-degree-of-freedom vibratory system, with natural frequency q, acted upon by the forcing function $S \sin (\omega t + \phi)$, while m constitutes an independent mass system acted upon by the constant force $-w$. The equations of motion for this condition are:

$$M\ddot{x} + kx = kS \sin (\omega t + \phi) \tag{1}$$

$$m\ddot{x} = -w. \tag{2}$$

(2) *Connected motion.* If the velocities of M and m are equal, k and $(M + m)$ constitute a single-degree-of-freedom vibratory system with natural frequency p, acted upon by $S \sin (\omega t + \phi)$ and $-w$. The equation of this motion is:

$$(M + m)\ddot{x} + kx = kS \sin (\omega t + \phi) - w. \tag{3}$$

The behavior of the system is governed by the above equations and by the following considerations from which the boundary conditions will be determined: (1) Connected motion will *begin* whenever the velocity of M overtakes that of m. Since the velocity of m cannot be negative,

the velocity of M must be *positive* for this to occur. (2) Connected motion will *end* if (*a*) the positive velocities of both masses drop to zero and if that of M becomes negative, or (*b*) if the positive velocity of M becomes less than that of m.

The Four Classes of Motion

A qualitative idea of the four classes of steady state motion will now be presented before a quantitative solution is entered into. Figures 2, 3, 4 and 5 show qualitative diagrams of mass, displacement, velocity and

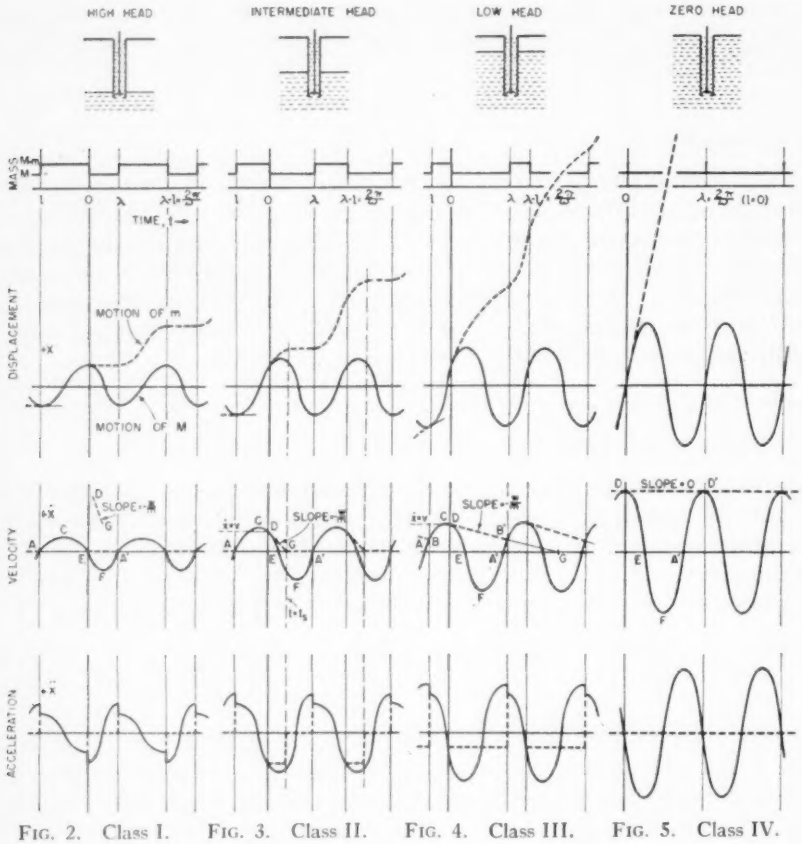

FIG. 2. Class I. FIG. 3. Class II. FIG. 4. Class III. FIG. 5. Class IV.

Qualitative diagrams of the four classes of motion.

acceleration as functions of time. For purpose of illustration, the Classes have been related to difference in pumping head; it is assumed that all other parameters are constant. If the pumping head is high the simple Class I Motion may be expected; if it is zero the trivial case of Class IV

will result. It will appear later that change in Class is also related to the amplitude S of the forcing motion and to the forcing frequency ω.

Consider the velocity diagram of Fig. 3. At point A, m is at rest and the velocity of M is passing through zero; consequently, connected motion begins at this point. Between points A and C the two masses move together with increasing velocity, and between C and D they continue to move together but with decreasing velocity. At D the straight

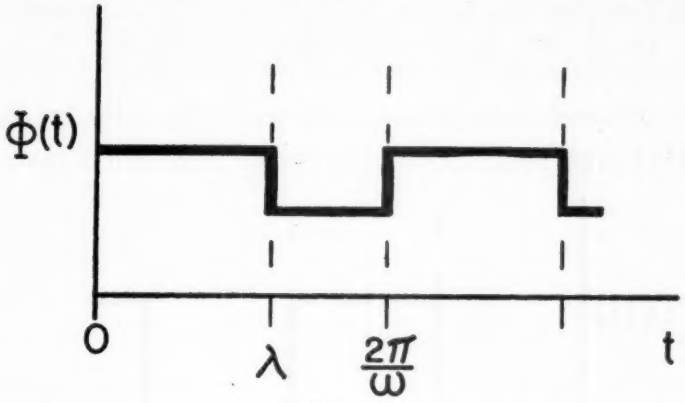

FIG. 6.

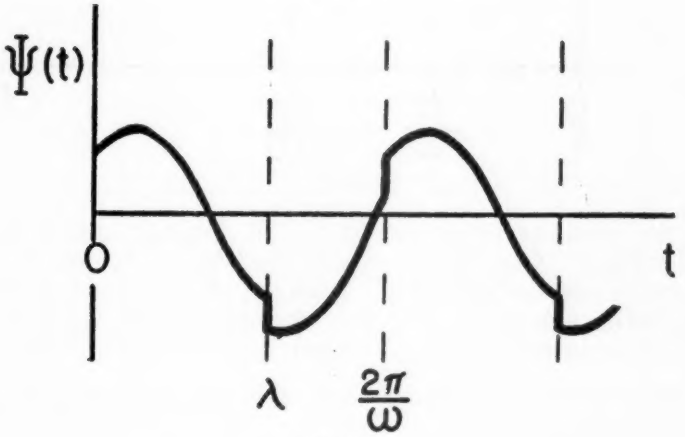

FIG. 7.

line with slope $-\frac{w}{m}$ (Eq. 2), representing the limiting negative acceleration of m, is tangent to the velocity diagram. At the point of tangency the connection between M and m ceases. The mass m will then continue to move with uniformly decreasing positive velocity until it stops at G; it cannot acquire a negative velocity because the accelerating

force $-w$ is now opposed by the action of the standing valve (or by the corresponding stop in some analogous system). During the subsequent interval of disconnected motion, M moves with a velocity indicated by D, E, F, A' until at A' it again connects with m by the action of the traveling valve; the cycle is then repeated. Call this Class II Motion.

If the slope $-\frac{w}{m}$ is *smaller* than the slope of any part of the curve A, C, E, then there is no point at which the line DG can be tangent to the curve. In this case the velocity diagram becomes that of Fig. 2 and Class I Motion is obtained, the commonly assumed type. If the slope $-\frac{w}{m}$ is sufficiently *larger* than that shown in Fig. 3, *but less than zero*, Class III Motion results as shown in Fig. 4. Note that in this case the

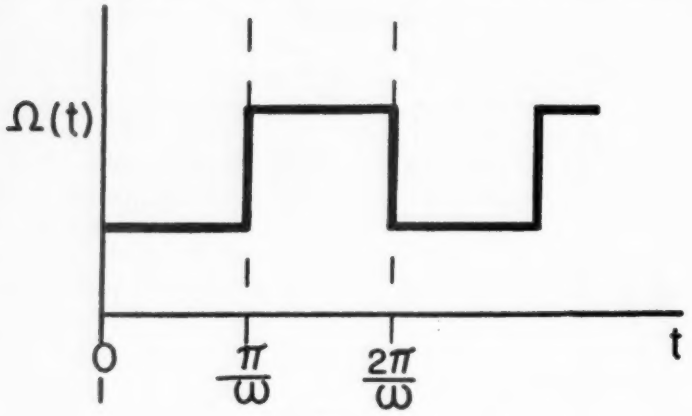

FIG. 8.

secondary inertia never comes to rest. The limiting case of Class IV is shown in Fig. 5. Once the steady state has been reached the two masses move independently, m having a constant velocity equal to the maximum positive velocity of M. It is obvious that the Class of Motion cannot be accurately predicted without a complete investigation since the Class depends not only on $\frac{w}{m}$ but also on the velocities of the system.

The Meissner Equation

The differential equations of motion (1) and (3) can be written as a single equation,

$$\ddot{x} + \Phi(t)x = \Psi(t), \tag{1a}$$

where $\Phi(t)$ and $\Psi(t)$ are periodic functions of t whose forms are given by

Figs. 6 and 7. If Eq. 1*a* be compared with the well known Meissner[4] equation:

$$\ddot{x} + \Omega(t)x = 0, \tag{1b}$$

where $\Omega(t)$ is a periodic function of the form given in Fig. 8, it can be said that Eq. 1*a* is a non-homogeneous generalized Meissner Equation. An important feature of Eq. 1*a* is that the boundary value λ (Fig. 6), which in the case of Eq. 1*b* is equal to $\frac{\pi}{\omega}$, in our case is not known and has to be determined together with the solution of (1*a*).

Class I Solution

Equation 3 is valid between points A and E of Fig. 2 and Eq. 1 between points E and A'.

Connected motion

Solution of (3) gives:

$$x = A \cos pt + B \sin pt + SP \sin (\omega t + \phi) - \delta. \tag{4}$$

The boundary conditions at $t = 0$, $x = x_1$, $\dot{x} = 0$ determine A and B so that:

$$x = [x_1 - SP \sin \phi + \delta] \cos pt - SP \frac{\omega}{p} \cos \phi \cdot \sin pt + SP \sin (\omega t + \phi) - \delta, \tag{5}$$

where $l \gtrless t \gtrless 0$. Moreover, at $t = l$, $x = x_2$ and $\dot{x} = 0$.

Disconnected motion

Solution of (1) gives:

$$x = C \cos qt + D \sin qt + SQ \sin (\omega t + \phi). \tag{6}$$

The boundary conditions at $t = 0$, $x = x_1$, $\dot{x} = 0$, determine C and D so that:

$$x = [x_1 - SQ \sin \phi] \cos qt - SQ \frac{\omega}{q} \cos \phi \cdot \sin qt + SQ \sin (\omega t + \phi), \tag{7}$$

where $0 \gtrless t \gtrless \lambda$. Moreover, at $t = \lambda$, $x = x_2$ and $\dot{x} = 0$.

By means of Eqs. 5 and 7 and the boundary conditions at $t = \lambda$ and $t = l$, four equations in x_1, x_2, ϕ and $\lambda = l + \frac{2\pi}{\omega}$ are obtained. These

[4] E. MEISSNER, "Schweizerische Bauzeitung," Vol. 72, 1918, pp. 95–99.

equations can be transformed into the following four:

$$x_1 = SQ\left\{\sin\phi - \frac{\omega\left[\cos\phi\cdot\cos q\lambda - \cos(\omega\lambda+\phi)\right]}{q\sin q\lambda}\right\} \tag{8}$$

$$x_2 = SQ\left\{\sin(\omega\lambda+\phi) - \frac{\omega\left[\cos\phi - \cos(\omega\lambda+\phi)\cdot\cos q\lambda\right]}{q\sin q\lambda}\right\} \tag{9}$$

$$\left\{P\left[1 - \frac{\omega\sin\omega l}{p\sin pl}\right] - Q\left[1 - \frac{\omega\sin\omega\lambda}{q\sin q\lambda}\right]\right\}\sin\phi + \left\{\frac{P\omega(\cos\omega l - \cos pl)}{p\sin pl} - \frac{Q\omega(\cos\omega\lambda - \cos q\lambda)}{q\sin q\lambda}\right\}\cos\phi - \frac{\delta}{S} = 0 \tag{10}$$

$$\left\{P\left[\cos\omega l - \frac{\omega\sin\omega l}{p\tan pl}\right] - Q\left[\cos\omega\lambda - \frac{\omega\sin\omega\lambda}{q\tan q\lambda}\right]\right\}\sin\phi + \left\{P\left[\sin\omega l - \frac{\omega(1-\cos\omega l\cdot\cos pl)}{p\sin pl}\right] - Q\left[\sin\omega\lambda - \frac{\omega(1-\cos\omega\lambda\cdot\cos q\lambda)}{q\sin q\lambda}\right]\right\}\cos\phi - \frac{\delta}{S} = 0. \tag{11}$$

ϕ and λ or l can be found by trial solution of the simultaneous Eqs. 10 and 11. This is at best a very laborious process, but can be expedited by graphing the individual equations as shown in Fig. 9. (Actually, it

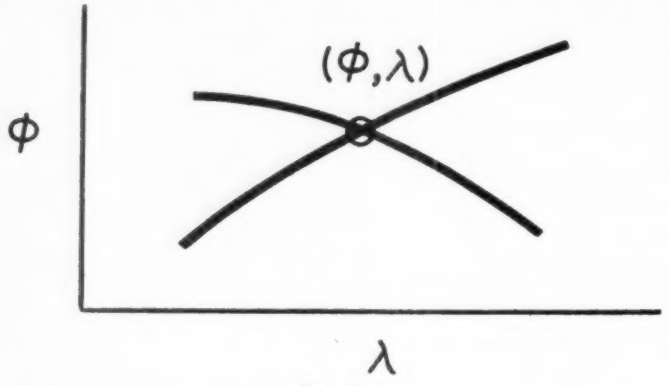

FIG. 9.

is more convenient to carry out the numerical work in terms of $\omega\lambda$ or ωl rather than λ.) It also will be found helpful to graph the coefficients of the sin ϕ and cos ϕ terms as functions of λ. The latter graph allows interpolation for the coefficients and assists in locating errors.

When ϕ, λ, x_1 and x_2 have been found, their substitution into (5) and (7) will give explicit expressions for x. The remaining problem, for a

given set of parameters, is to verify that the original assumption of Class I Motion is correct. In order for the motion to be of Class I the slope $-\frac{w}{m}$ must be smaller than the slope of any part of the velocity curve A, C, E of Fig. 2, that is, the following relationship must hold:

$$\ddot{x}_{\min} > -\frac{w}{m}, \quad \text{where } l \gtrless t \gtrless 0. \tag{12}$$

If the original assumption proves to be incorrect then it is assumed that the motion is either of Class II or of Class III and a new solution must be made.

The steady-state magnification factor is

$$\mathfrak{M} = \frac{x_1 - x_2}{2S}. \tag{13}$$

Class II Solution

The solution for Class II (and also for Class III) is of the same general form as for Class I but is somewhat more complicated. Equation 3 is valid between points A and D of Fig. 3 and Eq. 1 between D and A'. Equation 2 holds only during the time inteval between D and G since at G the motion of m is arrested by the "standing valve." The solutions of Eqs. 3 and 1 are the same as for Class I (see Eqs. 4 and 6) except that the boundary conditions, and hence the constants A, B, C and D, will be different.

Connected motion

The boundary conditions at $t = 0$, $x = x_1$, $\dot{x} = v$, $\ddot{x} = -\frac{w}{m}$, will determine A and B so that:

$$x = \left[\delta R - SP\frac{\omega^2}{p^2}\sin\phi\right]\cos pt + \left[\frac{v}{p} - SP\frac{\omega}{p}\cos\phi\right]\sin pt + SP\sin(\omega t + \phi) - \delta, \tag{14}$$

where $l \gtrless t \gtrless 0$, and

$$x_1 = S\sin\phi + \delta R\frac{p^2}{q^2}. \tag{14a}$$

Moreover, at $t = l$, $x = x_2$ and $\dot{x} = 0$.

Disconnected motion

The boundary conditions at $t = 0$, $x = x_1$, $\dot{x} = v$, will determine C and D so that:

$$x = [x_1 - SQ\sin\phi]\cos qt + \left[\frac{v}{q} - SQ\frac{\omega}{q}\cos\phi\right]\sin qt + SQ\sin(\omega t + \phi), \tag{15}$$

where $0 \gtrless t \gtrless \lambda$. Moreover, at $t = \lambda$, $x = x_2$ and $\dot{x} = 0$.

By introducing the boundary conditions and Eq. 14*a* for x_1, into Eqs. 14 and 15, we obtain four equations in x_2, v, ϕ and λ. The equations can be transformed, after considerable manipulation, into the following four:

$$v = \delta Rp \tan pl - SP\omega\left[\frac{\omega \sin \phi \cdot \tan pl}{p} - \cos \phi + \frac{\cos (\omega l + \phi)}{\cos pl}\right] \quad (16)$$

$$x_2 = \delta\left[\frac{R}{\cos pl} - 1\right] - SP\left[\frac{\omega^2 \sin \phi}{p^2 \cos pl} + \frac{\omega \cos (\omega l + \phi) \cdot \tan pl}{p} - \sin (\omega l + \phi)\right] \quad (17)$$

$$\left\{P \cos q\lambda\left[\sin \omega l - \frac{\omega}{p} \sin pl\right] - Q \cos pl\left[\sin \omega\lambda - \frac{\omega}{q} \sin q\lambda\right]\right\} \sin \phi + \{P \cos q\lambda\,[\cos pl - \cos \omega l] - Q \cos pl\,[\cos q\lambda - \cos \omega\lambda]\} \cos \phi + \frac{\delta Rp}{S\omega}\left\{\sin pl \cdot \cos q\lambda - \frac{p}{q} \sin q\lambda \cdot \cos pl\right\} = 0 \quad (18)$$

$$\left\{P \cos q\lambda\left[\frac{\omega}{p} \sin \omega l \cdot \sin pl + \cos \omega l \cdot \cos pl - \frac{\omega^2}{p^2}\right] - Q \cos pl\left[\frac{\omega}{q} \sin \omega\lambda \cdot \sin q\lambda + \cos \omega\lambda \cdot \cos q\lambda - \frac{\omega^2}{q^2}\right]\right\} \sin \phi + \left\{P \cos q\lambda\left[\sin \omega l \cdot \cos pl - \frac{\omega}{p} \cos \omega l \cdot \sin pl\right] - Q \cos pl\left[\sin \omega\lambda \cdot \cos q\lambda - \frac{\omega}{q} \cos \omega\lambda \cdot \sin q\lambda\right]\right\} \cos \phi + \frac{\delta}{S}\left\{R\left[\cos q\lambda - \frac{p^2}{q^2} \cos pl\right] - \cos pl \cdot \cos q\lambda\right\} = 0. \quad (19)$$

The remaining problem is similar to that for Class I, but the assumption of Class II Motion must be verified as follows. Referring to the velocity diagram of Fig. 3 it can be seen that the line DG must intersect the time axis between E and A'; the following relationship, therefore, must be satisfied:

$$0 < v < \frac{w}{m}\lambda. \quad (20)$$

The steady-state magnification factor is

$$\mathfrak{M} = \frac{x_{max} - x_2}{2S}. \quad (21)$$

Class III Solution

Equation 3 is valid between points B and D of Fig. 4 and Eqs. 1 and 2 between D and B'.

Connected motion

Boundary conditions: at $t = 0$, $x = x_1$, $\dot{x} = v$, $\ddot{x} = -\frac{w}{m}$; at $t = l$, $x = x_2$, $\dot{x} = v - \frac{w}{m}\lambda$. The boundary conditions at $t = 0$ are the same as for Class II; therefore, Eqs. 14 and 14*a* for x and x_1 apply here as well.

Disconnected motion

Boundary conditions: at $t = 0$, $x = x_1$, $\dot{x} = v$; at $t = \lambda$, $x = x_2$, $\dot{x} = v - \frac{w}{m}$. x is given by Eq. 15.

By methods similar to those for Class II the following equations are obtained:

$$v = \frac{\delta R p[p\lambda - \sin pl] + SP\omega\left[\frac{\omega}{p}\sin\phi\cdot\sin pl - \cos\phi\cdot\cos pl + \cos(\omega l + \phi)\right]}{1 - \cos pl} \tag{22}$$

$$x_2 = \delta\left\{R\left[\frac{p\lambda\sin pl}{1-\cos pl} - 1\right] - 1\right\} + SP\left\{\frac{\omega^2\sin\phi}{p^2} + \frac{\omega\sin pl\,[\cos(\omega l+\phi) - \cos\phi]}{p(1-\cos pl)} + \sin(\omega l + \phi)\right\} \tag{23}$$

$$\left\{P\frac{\omega}{p}[1-\cos q\lambda]\left[\frac{\omega}{p}\sin pl - \sin\omega l\right] + \frac{q}{p}\left[1-\cos pl\right]\times\left\{Q\left[\frac{\omega}{q}\sin\omega\lambda - \sin q\lambda\right] + \sin q\lambda\right\}\right\}\sin\phi$$
$$+\frac{\omega}{p}\left\{P[1-\cos q\lambda][\cos\omega l - \cos pl] + Q[1-\cos pl][\cos q\lambda - \cos\omega\lambda]\right\}\cos\phi$$
$$+\frac{\delta R}{S}\left\{[1-\cos q\lambda][p\lambda - \sin pl] - [1-\cos pl]\left[p\lambda - \frac{p}{q}\sin q\lambda\right]\right\} = 0 \tag{24}$$

$$\left\{P\left[1+\cos\omega l - \frac{\omega\sin\omega l\cdot\sin pl}{p(1-\cos pl)}\right] - Q\left[1+\cos\omega\lambda - \frac{\omega\sin\omega\lambda\cdot\sin q\lambda}{q(1-\cos q\lambda)}\right]\right\}\sin\phi$$
$$+\left\{P\left[\sin\omega l + \frac{\omega\sin pl(\cos\omega l - 1)}{p(1-\cos pl)}\right] - Q\left[\sin\omega\lambda + \frac{\omega\sin q\lambda(\cos\omega\lambda - 1)}{q(1-\cos q\lambda)}\right]\right\}\cos\phi$$
$$+\frac{\delta}{S}\left\{Rp\lambda\left[\frac{\sin pl}{1-\cos pl} - \frac{p\sin q\lambda}{q(1-\cos q\lambda)}\right] - 2\right\} = 0. \tag{25}$$

The steady-state magnification factor is

$$\mathfrak{M} = \frac{x_{max} - x_{min}}{2S}. \tag{26}$$

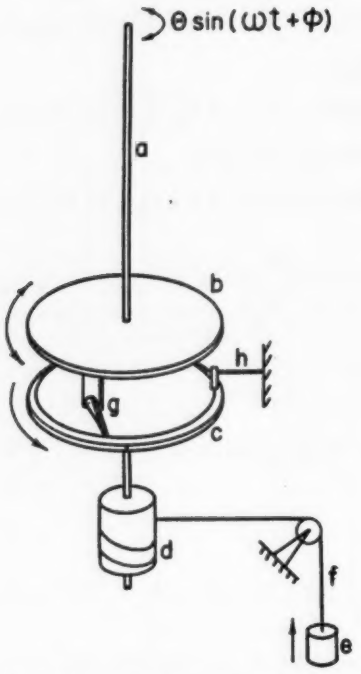

FIG. 10(*a*). Schematic diagram of mechanical analogue.

a = torsional restoring element of negligible inertia.
a' = chuck for attaching restoring element to driving head.
a'' = chuck and differential screw for attaching restoring element to primary inertia.
b = primary inertia; upper disc and pawls *g*.
c = principal part of secondary inertia (lower disc).
d = hoisting drum attached to *c*.
e = gravity load.
f = hoisting string of large longitudinal stiffness.
g = three upper pawls; the pawls are hinged to the upper disc and engage a soft metallic ring set into the lower disc.
h = three lower pawls which prevent negative rotation of the secondary inertia.
h' = supports on which pawls *h* are hinged.
i = adjustable speed d.c. motor; tachometer not shown.
j = armature and field rheostats.
k = vee-belt drive; gear reducer for very low speeds not shown.
l = adjustable radius crank.
m = connecting rod from crank to cross-arm.
n = cross-arm attached to driving head.
o = recording mechanism, showing paper drive and pen for recording angular displacement of upper disc (piston motion).
p = electro-magnetic device and attached pen for indicating half-cycle intervals of driving motion and each one-sixth revolution of lower disc.

The system *a*, *b* represents the piston rod and piston (Fig. 1); the system *c*, *d*, *e*, *f* correspond to the fluid column The pawls *g* represent the traveling valve, and the pawls *h*, the standing valve.

Class IV Solution

The steady-state solution is simple and will not be given. The motion results when $\frac{\delta}{S} \to 0$.

It is important to notice that in *all Classes of Motion*, x is a linear function of the forcing amplitude S provided that the term $\frac{\delta}{S}$ is constant.

EXPERIMENTAL APPARATUS

It is obvious that calculations based on the theoretical solutions will be extremely laborious. A mechanical apparatus has therefore been

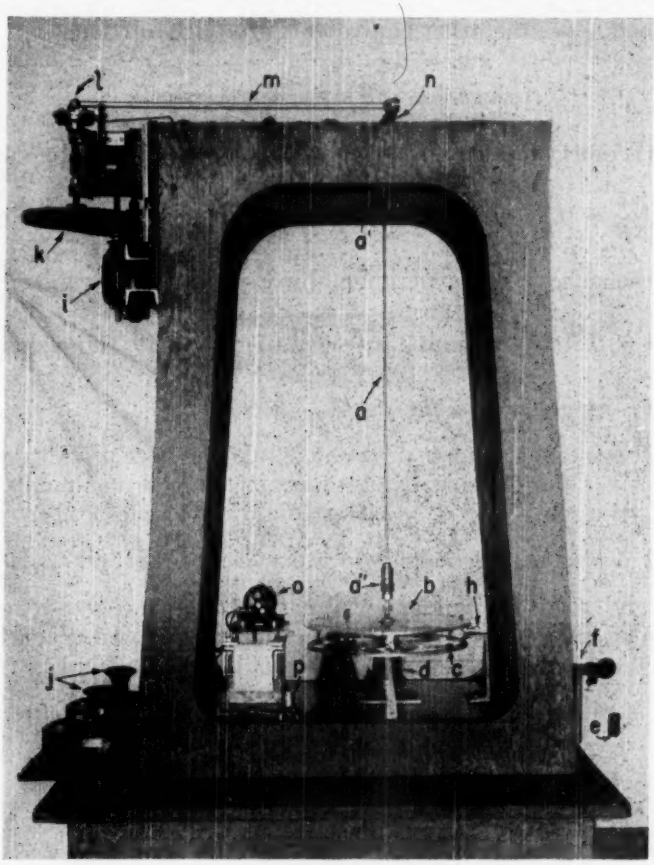

FIG. 10(*b*). Photograph of experimental apparatus. (For explanation of markings, see under Fig. 10(*a*).)

set up as a special "computing machine" for the solutionof the equations. For this purpose a rotational, rather than translational, system was found desirable (Fig. 10). Considerable care was taken to reduce damping to a nearly negligible factor and to prevent looseness in the connecting rod and crank bearings. The damping was mainly of the Coulomb type. The driving mechanism was statically balanced and there was no significant vibration caused by the drive.

The differential equations of motion of the analogue can be shown to be identical in form to those for the idealized pump. The constant force w, appearing in Eqs. 2 and 3, has as its equivalent a constant moment $w'r$ where w' is the weight of the suspended gravity load e and r is the radius of the hoisting drum d; the mass m becomes a moment of inertia j where j is composed of two parts, the moment of inertia of the lower disc c and drum d plus the equivalent moment of inertia of the suspended load, $\frac{w'r^2}{g}$; the other equivalents are fairly obvious.

THEORETICAL AND EXPERIMENTAL RESULTS

The theoretical calculations were limited mainly to the case, $\frac{q}{p} = 1.25$, and $\frac{\delta}{S} = 0.5$; values of $\frac{\omega}{p}$ ranged from 0 to 1.75, chiefly in the region of resonance. The experimental work covered the cases: $\frac{q}{p} = 1.25$ and $\frac{\delta}{S} = 0, 0.5, 1.0, 1.5$ and 2.0; $\frac{q}{p} = 1.67$, $\frac{\delta}{S} = 0.5$. The experimental values of $\frac{\omega}{p}$ ranged from near 0 to 3.

The above dimensionless parameters take on more practical meaning if we consider the dimensions of an actual oil well and pump. For a 3500-ft. string of $\frac{3}{4}$-in. rod, a 2250-ft. fluid head, and a $1\frac{1}{4}$-in. plunger, $\frac{q}{2\pi} \approx 1.2$ cycles per second, $\frac{p}{2\pi} \approx 1.0$ cycle per second and $\frac{q}{p} = 1.2$, where q and p are the fundamental natural frequencies of longitudinal vibration of, respectively, the rod alone, and the rod with a concentrated load at the end equal to the total weight of the fluid column. If there is a 20-in. stroke at the surface ($S = 10$ in.), $\frac{\delta}{S} = 0.21$. If the pump is operating at, say, 20 strokes per minute, $\frac{\omega}{p} = 0.33$.

The Particular System: $\frac{q}{p} = 1.25, \frac{\delta}{S} = 0.5$

Figure 11 shows two examples of the calculated motion of the system; the variables are $\frac{x}{S}$ and ωt. The displacement of m at $t = 0$ has been taken arbitrarily as equal to $\frac{x_1}{S}$. The term, slip, refers to the overtravel of the mass m during one cycle. We will call this *positive slip*, or simply *slip*, to distinguish it from an entirely different negative slip

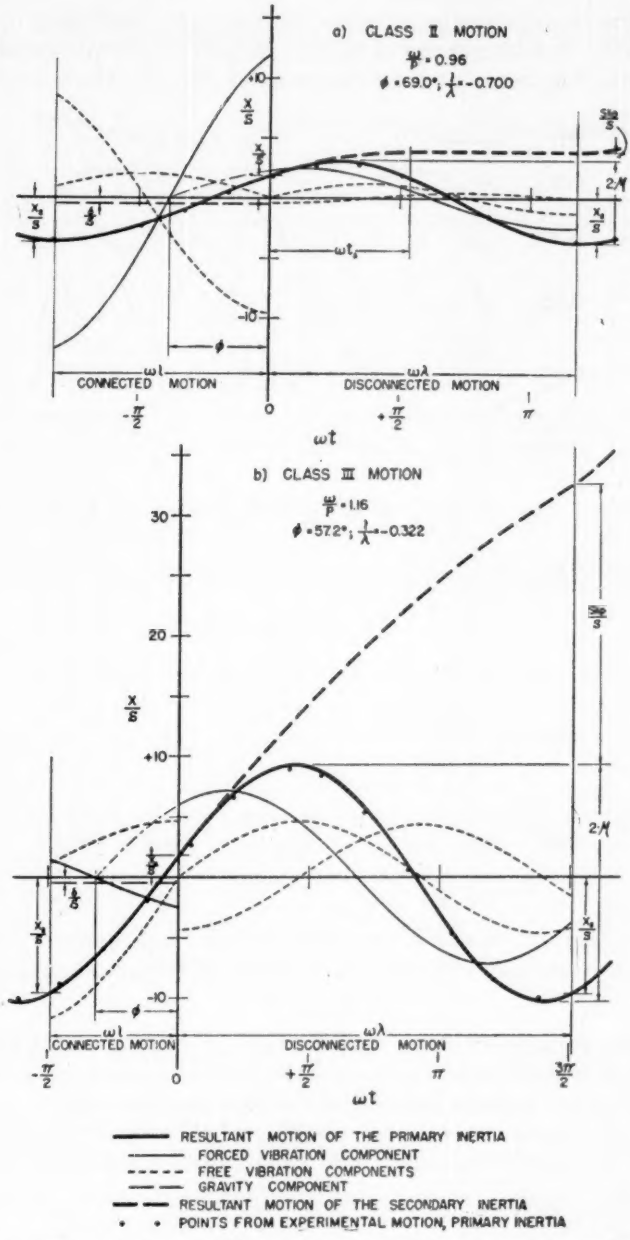

FIG. 11. Theoretical examples of motion with comparative experimental points.

$\frac{q}{p} = 1.25; \frac{\delta}{S} = 0.5.$

which is due to experimental causes, for example, inefficient operation of the pawls. A wider range of forcing frequency is represented in the experimental displacement-time diagrams of Fig. 12, where we see that

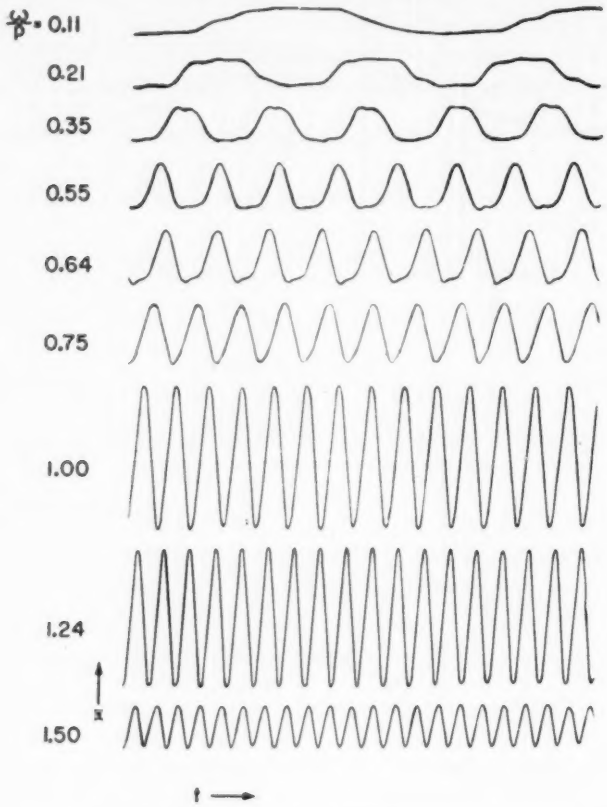

FIG. 12. Samples of experimental displacement-time records of the motion of the primary inertia.

$\frac{q}{p} = 1.25; \frac{\delta}{S} = 0.5.$

under some conditions there may be two or more parts of connected motion and two or more parts of disconnected motion in each cycle.[5] In this event the present theoretical solution does not hold. A solution to cover the more general case could probably be developed, but it would be extremely tedious. The calculations have been limited to values of $\frac{\omega}{p} > 0.65$.

[5] A similar situation arises in the problem of the steady forced vibration of a system with Coulomb damping. See J. P. DEN HARTOG, "Forced Vibrations with Combined Coulomb and Viscous Friction," *Trans. A.S.M.E.*, Vol. 53, 1931, APM-53-9-107.

In Fig. 13(*a*) the calculated ratio $\frac{l}{\lambda}$, meaning duration of connected

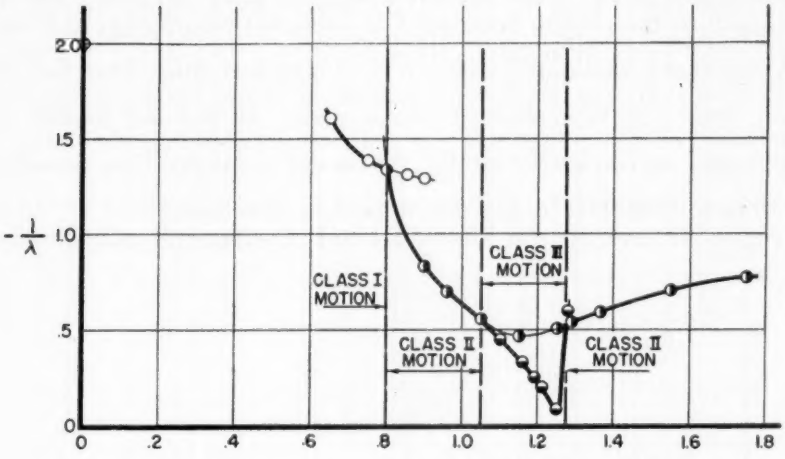

FIG. 13(*a*). Theoretical values of the ratio $\frac{\text{Duration of connected motion, } l}{\text{Duration of disconnected motion, } \lambda}$.

$\frac{q}{p} = 1.25; \frac{\delta}{S} = 0.5.$

The region $0 < \frac{\omega}{p} < 0.65$ was not explored for reasons explained in connection with Fig. 12.

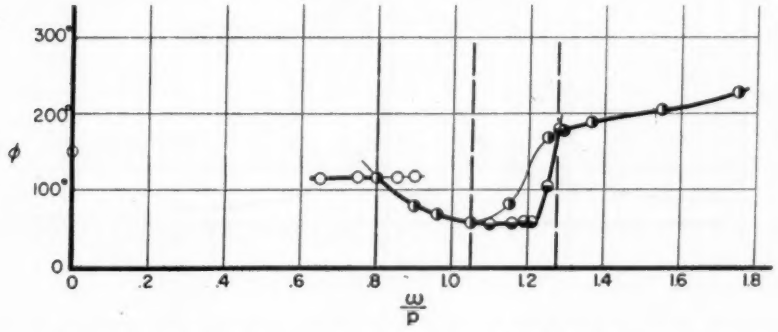

FIG. 13(*b*). Theoretical values of the phase angle ϕ of the forcing function.

$\frac{q}{p} = 1.25; \frac{\delta}{S} = 0.5.$

motion divided by duration of disconnected motion, has been plotted as a function of $\frac{\omega}{p}$; the phase angle ϕ appears in Fig. 13(*b*). The discontinuities in slope are due to the changes in the Class of Motion. It will be remembered that $\frac{l}{\lambda}$ and ϕ must be computed before the verification

of the Class of Motion can be made (Eq. 12 or 20). The points which are shown on the fine extended lines represent false solutions. They are real solutions as far as the equations in ϕ and λ are concerned, but do not apply to the motion because of an incorrect assumption of Class. The values of $\frac{l}{\lambda}$ and ϕ at $\frac{\omega}{p} \to 0$ can be determined either from the dynamic theory or from static considerations. It was not feasible to determine $\frac{l}{\lambda}$ and ϕ experimentally; the reason is obvious if one considers the difficulty, implied by Fig. 11, in locating zero time.

Figure 14 presents the theoretical and experimental magnification

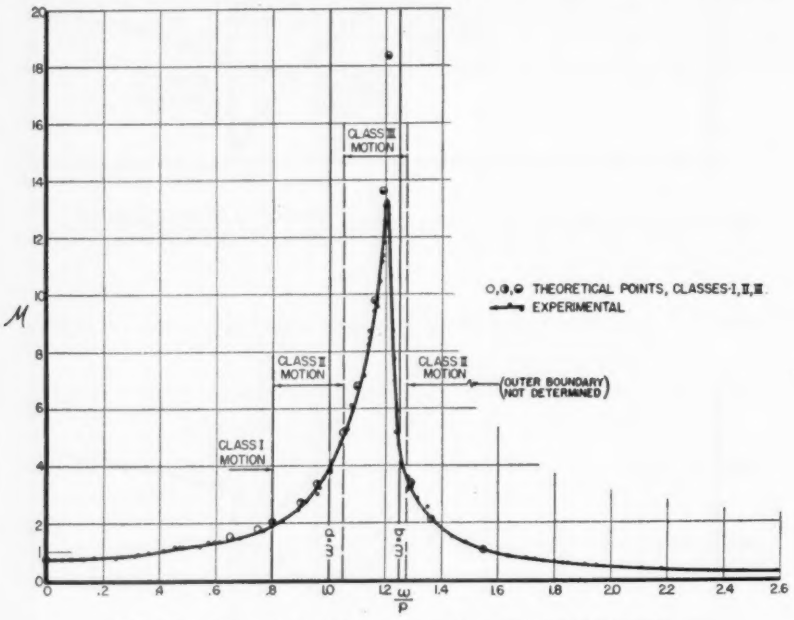

FIG. 14. Theoretical and experimental magnification factors.

$\frac{q}{p} = 1.25; \frac{\delta}{S} = 0.5.$

factors $\mathfrak{M}$ as functions of $\frac{\omega}{p}$. At $\frac{\omega}{p} \to 0$, $\mathfrak{M} < 1$; this is due to the influence of the constant active force w. The theoretical magnification factor for this case is,

$$\mathfrak{M} = 1 - \frac{1}{2}\frac{\delta}{S}. \tag{27}$$

There is one resonance peak; it lies between the points $\omega = p$ and $\omega = q$. There is no indication, either theoretically or experimentally, of any secondary resonances.

The Family of Systems: $\frac{q}{p} = 1.25$; $\frac{\delta}{S} = 0, 0.5, 1.0, 1.5, 2.0$; *Fig. 15*

The following items should be noted: (1) A limiting condition is given by $\frac{\delta}{S} \to 0$. The motion is Class IV regardless of the value of $\frac{\omega}{p}$ and the theoretical resonance diagram is identical with that for a linear system with zero damping factor. (2) The case $\frac{\delta}{S} = 0.5$ includes motions of Classes I, II and III, depending on $\frac{\omega}{p}$. The resonance peak lies

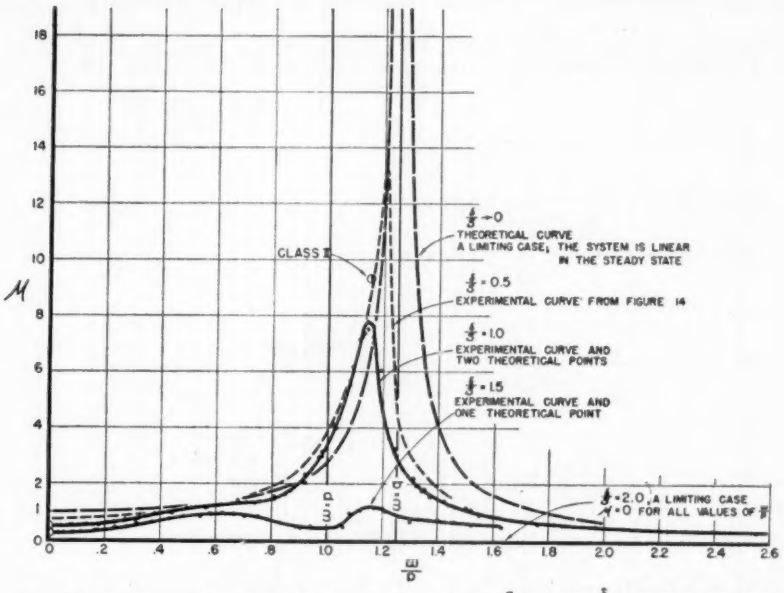

FIG. 15. Magnification factors for the family of systems: $\frac{q}{p} = 1.25$; $\frac{\delta}{S} = 0, 0.5, 1.0, 1.5, 2.0$.

in a region of Class III Motion. The theoretical and experimental points have already been shown in Fig. 14 and for clarity have been omitted here. (3) Since only two theoretical points were calculated for the case $\frac{\delta}{S} = 1.0$, the boundaries between the Classes of Motion cannot be established. However, it is reasonably certain that motions of only Classes I and II exist since the calculated point at $\frac{\omega}{p} = 1.15$ indicates Class II Motion at resonance. (4) Results for the case, $\frac{\delta}{S} = 1.5$, are entirely experimental except for the theoretical point at $\frac{\omega}{p} \to 0$. This

case represents a condition in which the static deflection caused by the unbalanced load is *greater* than the forcing amplitude; the response is greatly reduced over that for the case $\frac{\delta}{S} = 1.0$. (5) According to Eq. 27 when $\frac{\delta}{S} = 2.0$ the magnification factor is zero when $\frac{\omega}{p} \to 0$. It is apparent that the magnification factor must also be zero at other values of $\frac{\omega}{p}$. We thus have a limiting condition in which no motion occurs when the static deflection equals twice the forcing amplitude. The condition was tested experimentally and no perceptible motion occurred.

The Family of Systems: $\frac{\delta}{S} = 0.5$; $\frac{q}{p} = 1.00, 1.25, 1.67$; *Fig. 16*

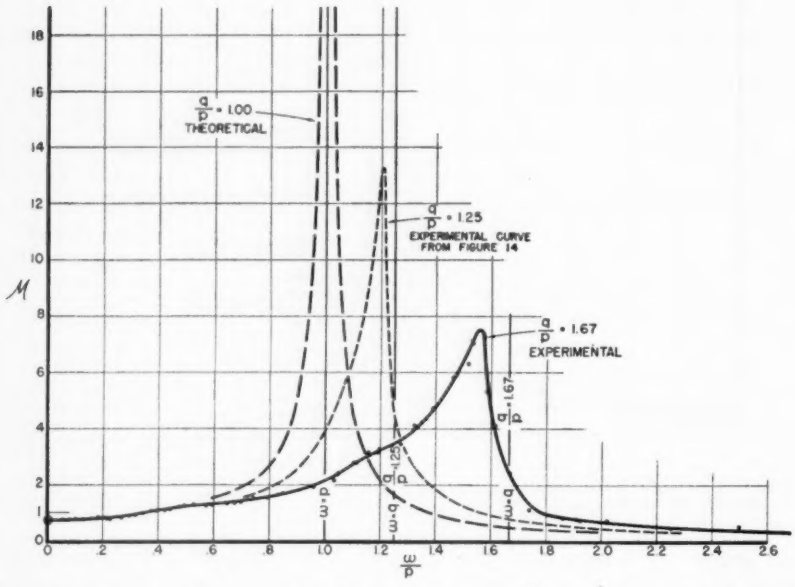

FIG. 16. Magnification factors for the family of systems: $\frac{\delta}{S} = 0.5$; $\frac{q}{p} = 1{,}00, 1.25, 1.67$.

The limiting case, $\frac{q}{p} = 1$, represents a system which is linear as far as mass is concerned, that is, there is no change in natural frequency, but which is still non-linear in a sense due to the existence of a constant retarding force during the connected part of the cycle. Such a system is in effect one with Coulomb damping which acts only when the velocity is positive. The differential equations of motion are given by Eqs. 1 and 3 if we let the secondary inertia m become zero and if we replace the

gravity load w by a constant force F. The motion is Class I and the solutions may be obtained by a simple revision of those for the more general case (the solutions have been checked by an independent derivation). The response at resonance, for $\frac{\delta}{S} = 0.5$, is infinite.[6] An increase in degree of nonlinearity, as defined by $\frac{q}{p}$, results in a lowering of the resonance magnification, as would be expected. As is anticipated by Eq. 27 the systems have the same magnification at $\frac{\omega}{p} \to 0$.

Positive Slip

Figure 17 shows experimental and theoretical slips for the system.

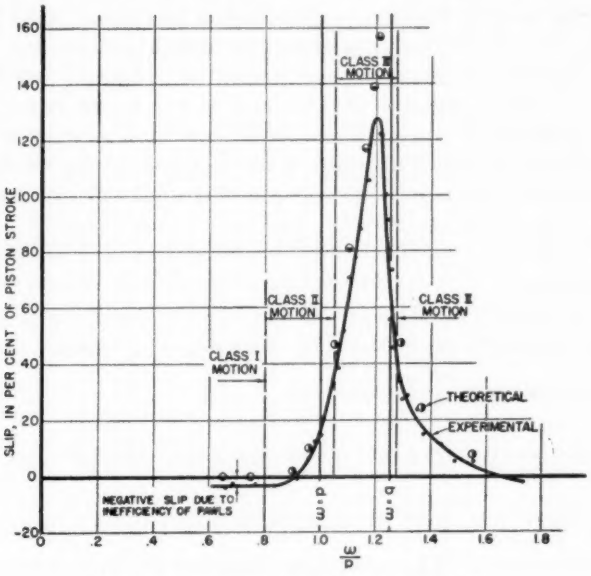

FIG. 17. Theoretical and experimental slip.
$\frac{q}{p} = 1.25; \frac{\delta}{S} = 0.5.$

$\frac{q}{p} = 1.25, \frac{\delta}{S} = 0.5$. The agreement between theory and experiment is good, particularly if the experimental curve is raised until no negative slip appears.

System with Distributed Parameters

A limited amount of experimental work was done with a system in

[6] Compare with conclusions reached by Den Hartog (see footnote 5) for the case of *symmetrical* Coulomb damping.

which the lumped inertia and elastic restoring element (*b* and *a* of Fig. 10) representing the piston rod were replaced by a helix having uniformly distributed elasticity and inertia. No attempt was made to replace the lumped inertia representing the fluid column with a distributed system. For forcing frequencies ranging from zero to near resonance with the fundamental mode, the system behaved in substantially the same manner as the lumped system, but with higher forcing frequencies the higher natural modes came into prominence.

SUMMARY AND CONCLUSIONS

(1) There are four distinct Classes of Motion, the determination of the Class depending on the parameters $\frac{\delta}{S}$, $\frac{\omega}{p}$ and $\frac{q}{p}$, that is, on the static deflection, the forcing amplitude, the forcing frequency and the natural frequencies. Class I Motion is the type usually assumed. Classes II and III are characterized by an over-travel or "slipping" of the secondary inertia. It is improbable that Class III ever occurs in a fluid pumping system although it may occur in some mechanical systems with low damping. Class II Motion involves relatively less slip than Class III and may possibly occur in some deep well reciprocating pumps. The existence of Class IV Motion requires that $\frac{\delta}{S} \to 0$; this Class is of no practical interest.

(2) In all Classes of Motion the displacement is a linear function of the forcing amplitude provided that the ratio of the static deflection to the forcing amplitude, $\frac{\delta}{S}$, is constant.

(3) Extensive calculations were made for the system, $\frac{q}{p} = 1.25$, $\frac{\delta}{S} = 0.5$. There was good agreement between theory and experiment except at resonance. The resonance diagram is similar in general appearance to that for a linear system with a moderately small damping characteristic. There is no evidence of more than one resonance peak, in spite of the fact that the system has two natural frequencies. The resonant frequency, or frequency of maximum response, lies between the natural frequencies.

(4) Variation in either of the parameters, $\frac{\delta}{S}$ or $\frac{q}{p}$, results in important changes in the resonance diagram. The limiting values of $\frac{\delta}{S}$ are 0 and 2. At $\frac{\delta}{S} \to 0$ the system becomes a linear one with theoretically

infinite response at resonance, occurring at $\omega = q$. As $\frac{\delta}{S}$ is increased the response and resonant frequency both decrease. At $\frac{\delta}{S} = 2.0$ the response is zero for all values of the forcing frequency.

(5) The natural frequency ratio $\frac{q}{p}$ has a lower limit of unity. At this value the system ceases to act as a pump, there is no change in mass, and the system becomes equivalent to one in which Coulomb damping acts only when the velocity is positive. As $\frac{q}{p}$ is increased, the resonant response decreases while the resonant frequency increases.

(6) It must be emphasized that the primary interest of the investigation has been the non-linearity of mass. The consequent minimizing of the effects of damping and throttling effect of the valves, and the lumping of the parameters, must be borne in mind.

ACKNOWLEDGMENT

This investigation is a part of a contract held by Stanford University and sponsored by the Office of Naval Research. The authors are indebted to Messrs. Robert Glasser, Chieh-Su Hsu, and Melvin Zaid, Graduate Students in Mechanical Engineering, for assistance in the calculations and in the experimental work.

Sound Waves Photographed (*Tele-Tech*, Vol. 9, No. 8).—A new technic recently developed at the Bell Telephone Laboratories makes actual photographs of sound wave patterns possible. The method will be used for studying the sound waves from telephone receivers and similar communications equipment, and for observing microwave radio wave patterns.

Equipment used in this work consists of a tiny microphone and a neon lamp, mounted on a swinging beam which scans the wave field. As the beam moves through the field, a clear picture of the sound radiation is built up by scanning, similar to the way in which television images are formed.

In one of the first experiments using the new technic, Bell scientists W. E. Kock and F. K. Harvey, who developed it, made photographs showing the precise acoustic effect of lenses. The lenses are similar to those used in microwave radio relay systems.

The microphone used in taking the pictures of sound is about the size of a quarter. It is mounted on the end of an aluminum arm four feet long along with a tiny, 110-volt neon lamp. A small loudspeaker radiating a sound wave is directed at one side of the acoustic lens. On the other side of the lens, the metal arm swings up and down, inscribing a three-foot arc through the path of the sound waves as they emerge from the focusing lens. Sound picked up by the microphone is carried to an audio-amplifier. As the sound level varies, the brightness of the neon lamp varies in proportion.

Viewed in a darkened room, the lamp glows brightly, then fades, then brightens again. As it traces its vertical pattern, it automatically moves horizontally away from the lens. Thus in a ten-minute time exposure, a sound wave pattern may be photographed.

Ambulance Planes (*American Aviation*, Vol. 14, No. 6).—More than 350 ambulance aircraft are available to doctors and hospitals throughout the nation, according to the "mercy aircraft" list published by CAA. The planes range in size from Cubs remodeled to carry one stretcher through four-place aircraft capable of carrying two stretchers and on up to converted C-47's.

The list gives the location and the operator's day and night phone numbers, type of plane, capacity to carry medical personnel and patients, and whether oxygen is available in flight. It is being sent to every county medical society so as to be within reach of the greatest number of doctors, and to a selected list of hospitals and clinics. Copies of this Directory of Ambulance Planes are available from CAA's Office of Aviation Information, Washington, D. C.

A prefabricated cold-weather building of stressed skin aluminum, containing its own heating and sanitation facilities, is being tested by the Army's Engineer Research and Development Laboratories at Fort Belvoir, Va. Designed to withstand 100-mph winds and temperatures as low as −65° F., it is made up of 8-ft. side, roof and floor panels with a honeycomb core, faced on both sides with an aluminum alloy. Primarily for troop use, the building has lightweight parts and is quickly erectable, even by relatively green troops.

THE DISTRIBUTION OF PARTICLE SIZES[1]

BY

F. KOTTLER[2]

SUMMARY

In this paper the older literature on distribution of particle sizes is reviewed critically. The distribution law should be connected with the law of growth of crystals, for which is chosen the exponential law as Galton indicated long ago in an analogous case. The law which then follows is the so-called Logarithmico-Normal law. In the application of that law to experimental data, graphical analysis by means of special charts is generally used. It is shown why this purely graphical method should be replaced by an algebraic one, such as is used, for instance, in psychophysics with a similar problem.

Part I

THE FACTS

It is a well-known fact that the sizes of small particles, for example, grains of sand, or the grains of silver halides in photography, are not uniform but differ, as a rule, from one particle to another. In Fig. 1 some photomicrographs are given to serve as an illustration. They belong to three photographic emulsions, Seed Lantern, High Speed B, and L-1, which will be considered later. The negatives of these photomicrographs were obtained with a microscope having a linear magnification of 2500. The prints which are reproduced here were then made by enlarging those negatives four times. The entire magnification is therefore 10,000 (1).[3]

A superficial inspection of Fig. 1 shows the variability of the sizes of the grains. A closer inspection reveals that there must be some law governing that variability. First, the sizes are confined within a certain *range*. Moreover, if we group together the particles in classes of magnitude, each *class* covering a certain *interval* of size, it will be found that some classes contain more grains than the others. As a rule, there will be a class which contains the largest number of particles. For all the other classes, there will be fewer particles, their numbers decreasing progressively towards the two ends of the range. The law which describes these facts is called, in mathematical statistics, a *frequency distribution law*. The problem before us is to find a mathematical expression which fits the facts as just described.

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[1] Communication No. 1269 from the Kodak Research Laboratories, Eastman Kodak Company, Rochester, N. Y.

[2] Kodak Park Works, Research Laboratories, Eastman Kodak Company, Rochester, N. Y.

[3] The boldface numbers in parentheses refer to the references appended to this part of the paper.

MEASUREMENT OF SIZE

In order to obtain the distribution law we must measure the size of each grain. However, such a measurement need not be very accurate; it merely serves to establish to which size class that grain belongs. Inaccuracy so introduced does not matter so long as the size of the grain is not near the chosen class-limits. Only for particles at the class-limits might there be some doubt as how to classify such grains and a

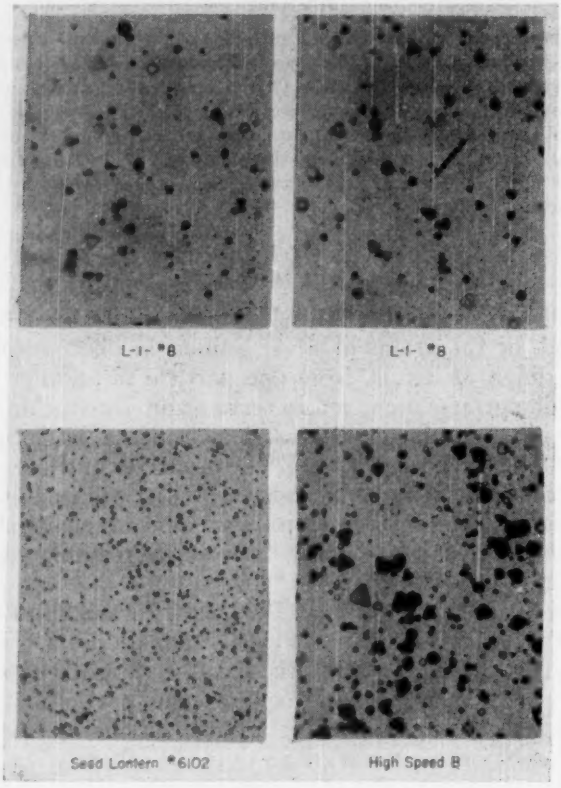

FIG. 1. Photomicrographs of three photographic emulsions. (Reduced one-half in reproduction.)

so-called *misclassification* may easily occur. However, if the *class-frequency*, that is, the number of particles belonging to a class, is large, the effect of such a misclassification will be small. The effect of misclassification will be more pronounced if the class-frequency is small, as necessarily is the case near the ends of the frequency range.

In classifying the grains as described above, it must be borne in mind that we do not see the grains themselves but only their images under the microscope. These images have only two dimensions. From the

present type of photomicrographs we learn nothing about the third dimension of the grains, and hence we do not know their real shape nor even which is their largest dimension.

The first to introduce the photomicrographic method in the classification of grain size of ZnO particles was H. Green, in 1921 (2). He used the *diameter* of each grain to measure its size. We need not go into his theoretical definitions of size by diameter. In practical work he usually took the diameters parallel to an arbitrarily fixed direction (the horizontal direction on a vertical projection screen). Such a measurement would hold if the particles were spheres, but in most cases, especially with the larger sizes, the particles cannot be likened to spheres. Without going into detail, it is obviously impossible to represent the size of a grain by means of any diameter without introducing the shape.

Sheppard and his collaborators therefore decided to use the *area* of the projection of the grain into the image plane of the microscope, the so-called *projected area*. (The expression "projective area" is also used but is inconsistent with the mathematical use of that term.) This area can be measured directly by a planimeter or more simply by using templates of adequate shapes and sizes to be applied to each grain image. The area can also be measured by taking two or more different diameters of each particle and determining what is called the "equivalent diameter." This diameter is defined as the diameter of a circle of the same area as the projected area.

The introduction of the projected area as a measure of size has several advantages: First, it is obviously independent of the shape.[4] Second, the projected area permits a direct application to some concepts of photographic theory, that is, the area blackened by the photographic effect. Finally, the projected area may be taken as a measure of the surface area of the grain.

This can be seen as follows: According to a theorem given long ago by the French mathematician, Cauchy (see a recent paper by Vouk (3)), the total surface of the grains would be four times their projected area, regardless of their shape, provided the particles are oriented at random. Actually, the grains are so oriented only in the initial state of a photographic emulsion. In a dried emulsion, there is considerable orientation. This introduces a change in the Cauchy factor and also a slight dependency upon the shape of the particles. It was found experimentally by Leermakers, Carroll, and Staud (4) in an example that the total surface was from 3.1 to 3.4 times the projected area, depending upon the shapes of the particles. In extreme cases, the factor might become less. In all cases, however, the ratio between total surface and total projected area appears to be approximately a constant for a given emulsion and practically independent of the shapes.

[4] The measurements of size used in the present paper do not differentiate with respect to shape.

This connection between projected area and surface will be of great importance in the theory presented in what follows. As we shall see, the determining factor in the growth of the size of a crystal is its surface. Thus, we can apply the results of the *statistical analysis* of the size distribution to the problem of the *growth of crystals*. We shall thus obtain, however, not the growth of an individual crystal but only the growth of an average crystal where the average is to be taken over the different directions of growth. Therefore, we shall have, so to speak, *isotropy* of growth with respect to all directions. Of course, it is well known that the individual crystals grow *anisotropically*, that is, differently with respect to the different crystal faces.

No mention of the *volume* has been made so far for reasons which should now be obvious. Several attempts to derive a mean value of the volume of the grains from the distribution of the projected areas have been found to be at variance with the mean value of the volume deduced experimentally from the total weight of the grains. This failure is understandable, since there is no connection between the projected area and the mean volume as the Cauchy relation between the projected area and the mean surface. It should be recalled that from microphotographs alone we do not know anything about the third dimension of the particles. However, this does not matter for our purpose of investigation since the volume plays no part in the law of growth.

THE NORMAL LAW AND WHY IT CANNOT BE A SIZE-DISTRIBUTION LAW

The statistical law most popular with physicists and chemists is the Gaussian Law of Errors or the "Normal Law." The first to apply it to the problem of the distribution of particle sizes was Green in the aforementioned paper. He was followed by Sheppard and his collaborators who applied it to the silver halides (5), and by Martin (6) who investigated crushed sand[5] (SiO_2). However, all these attempts to apply the Normal Law to that problem must fail, as we shall see in the sequel.

As early as 1879, Francis Galton (7), the founder of biometrics, warned against such an application ". . . which may lead to absurdities. [The Normal Law] asserts that deviations [from the mean] in excess must be balanced by deviations of equal magnitude in deficiency, therefore, if the former be greater than the mean itself, the latter must be less than zero, that is, must be negative. This is an impossibility in many cases." The Normal Law asserts therefore that "the existence of giants, whose height is more than double the mean height of their race, implies the possibility of the existence of dwarfs, whose stature is less than nothing at all."

Not all the numerous authors who have applied the Normal Law to problems of the height of human beings or more generally to the dis-

[5] Martin used also a microscopic method and measured sizes by diameters which were carefully defined in a statistical way.

tribution of size have been unaware of the absurdity described by Galton. Those who were aware of it tried to escape the difficulties due to the "negative" sizes, the existence of which is implied by the application of the Normal Law, in one of the following ways: Some tried to keep the frequencies of the very small sizes small; hence, the frequencies of the negative sizes beyond zero would be still smaller *a fortiori* and would so be negligible. However, this did not work, because the Normal Frequency Curve is symmetrical with respect to its peak, and hence the smallness of the frequencies near zero necessarily implies that the frequencies of the larger sizes beyond the mean will also be small. This, however, is contrary to the observations, which show that the empirical size-frequency curve possesses a long, right tail in the region of the larger sizes. This fact Sheppard and his collaborators discovered after many unsuccessful attempts to apply the Normal Law. In 1924, they wrote (8): "The [right] tail of the applied [Gaussian] curve was frequently too steep and short to fit the observed values."

Another way of dealing with the negative values which was practiced by some authors is as follows: The left tail of the Normal Curve is cut off at the size zero or just above it, and the negative and the zero values are "ignored." This would remove from the right tail the aforementioned handicap which results from the symmetry of the uncut Normal Curve. However, this procedure introduces other difficulties in place of the one difficulty that it removes. The law so obtained is no longer the Normal Law but is a "truncated" Normal Law, that is to say, one which has a reduced range. Instead of stretching from minus infinity to plus infinity, it extends only from some finite value to plus infinity. This introduces a new parameter, that is, the distance of this finite value from the mean, and so the "truncated" Normal Law is a law with three parameters, whereas the original Normal Law has merely two parameters, *viz.*, the mean and the standard deviation. It is incorrect to ignore the new parameter, so that in order to remove one difficulty we have introduced a graver difficulty by arriving at a more complicated form of a distribution law. The correct answer is, of course, to abandon any attempt to use the Normal Law for such purposes as size distribution, for which it was never intended by Gauss when he developed his theory of errors.

The use of a distribution law, not with its full range but only with a range cut down artificially, has been practiced by several authors in the field of size-distribution analysis. As has been already explained, this is a procedure by which practically nothing is gained. The aim should always be to find a law which fits the data over the entire range. Laws which fit only partially are simply not the correct laws and should be abandoned as quickly as possible. Since there may be some readers who still believe in the usefulness of the truncated range, we shall discuss in subsequent papers several "laws" which come under that head

and shall prove that they fit the data more poorly than other laws which fit over the entire range. Among those which will be treated in that manner are the attempts to apply the Normal Law to the problem of size distribution, as given in the aforementioned papers by Green, Sheppard, and Martin.

PHYSICOCHEMICAL ANALOGUES OF THE NORMAL LAW

There are some laws in physical chemistry which, because they resemble in their mathematical form the Gaussian Law of Errors, have been quoted in support of that law. First, there is Maxwell's Law of the distribution of the velocities of the molecules in a perfect gas. It is a three-dimensional generalization of Gauss's Law which the statisticians call "a trivariate normal distribution." This formal similarity was used by Sheppard (9) to deduce that the size distribution of particles must be normal. He assumes that with silver halide crystals the particles grow as a consequence of collisions with the ions of silver or halogen contained in the surrounding solution. Part of the colliding ions would become attached to some of the particles. Since the *velocities* of the colliding masses obey Maxwell's Law, the *sizes* would also obey what Sheppard calls "Maxwell's Law of Errors." However, distribution of sizes and distribution of velocities are different subjects. Moreover, Maxwell's Law can be applied only to a perfect gas or a dilute solution. It does not hold for a saturated solution such as the solution in which crystallization takes place.

Furthermore, it is obvious that collisions alone are insufficient to account for the growth of crystals. There are powerful electric forces between the particles and the ions which come into play within a certain proximity of both and which act so as to direct the ions towards the centers of attraction. It goes without saying that thereby the probability of an attachment is greatly increased, compared with the probability of an attachment following a collision according to Maxwell's Law.

Somewhat similar but more to the point is another argument which uses the law of diffusion of molecules or particles in solutions. As is well known, we obtain from Fick's equation a probability of the displacement from a mean position in any linear dimension which also has the form of Gauss's law. Now, it is well known that, according to theories given long ago by Noyes and Nernst (10), diffusion plays an important rôle in the process of crystallization. However, the same arguments apply here as above, and again the probability of an attachment due to electric forces by far outweighs the probability of an attachment due to diffusion alone.

Without going into further detail, it is obvious that Maxwell's and Fick's Laws have little connection with the mechanism underlying the growth of crystals. Their formal resemblance to the Normal Law can therefore contribute nothing to diminish the inconsistency introduced by the negative sizes implied in the application of that law.

THE EXPONENTIAL LAW OF GROWTH

In the paper already mentioned, Galton indicated the law of growth which should be assumed for the distribution of sizes so that it is free from the absurdity of the negative sizes. Quoting sociology as an example, he mentions "the law of ordinary increase as exemplified by the growth of a population, where an already large nation tends to become larger than a small one under similar circumstances or when a capital employed in a business increases in proportion to its size." This means that the rate of growth is proportional to the momentary value of the size, that is to say:

$$\frac{dx}{dt} = kx, \tag{1}$$

where x is the size, t is the time, and k is the *velocity constant of the growth.* This equation is immediately integrated by:

$$x = A \exp kt, \tag{2}$$

where A is a constant of integration. Equation 2 is called the exponential law of growth, or sometimes, especially by biologists, the logarithmic law of growth or the law of Malthus. It has the same mathematical form as the well-known law of compound interest in business arithmetic.

Among the many applications which that law has had we are here concerned with its application to the growth of crystals. We may assume that each ion lying in the surface of a crystalline particle attracts a certain number of ions from the surrounding solution. *If that number is the same for each of the surface ions,*[6] it follows that the number of ions attracted, say, in unit time, is proportional to the number of ions present in the surface. This number, on the other hand, will be proportional to the area of the surface and this leads immediately to Eq. 1. Incidentally, it follows that the adequate measure of size is the surface area. This is in agreement with the findings (11) of experiment which "indicates the existence of an absorption layer in which the essential process of crystallization occurs."

Equation 1 has several analogues in other fields of physical chemistry, provided we change the sign of k and write it in the form:[7]

$$\frac{dx}{dt} = -kx. \tag{1a}$$

In this form it is the equation of a chemical reaction of the *first order* (unimolecular reaction). The quantity x denotes the variable concentration of a substance which disappears during the reaction. Instead

[6] This is not strictly true, since the crystal growth is anisotropic. However, as we have already seen, we may assume isotropy, since we do not deal here with individual crystal faces but with the statistical average over all directions.

[7] There is no physicochemical analogue to Eq. 1 with a positive value of k, for reasons to be discussed later.

of that, we can also interpret x as the number of moles present at the time, t.

The equation for radioactive disintegration is also of the form (1a). There, x is the number of radioactive atoms present at the time, t, and the equation gives the number of atoms disintegrating during the time interval, dt.

A third application of Eq. 1a will be found below in the discussion of breakage.

In all these cases, we note that the relative *rate of change*,

$$\frac{dx}{x},$$

is independent of x and, if the time interval, dt, is kept constant, is a constant characteristic of the process of change (growth or decay, respectively). This permits an important interpretation: The relative rate of change is a ratio of two numbers. In the numerator, there is the number of individuals changing during the interval, dt. In the denominator, there is the total number of individuals present at the beginning of the time interval. This ratio may therefore be defined as the *probability of change*. The probability so defined is *independent of the absolute number of individuals present*.[8] We shall return to this point when discussing breakage.

THE LOGARITHMICO-NORMAL DISTRIBUTION

From Eq. 2 we have

$$t = -\frac{1}{k} \ln A + \frac{1}{k} \ln x,$$

where ln stands for the natural logarithm. Introducing two new constants,[9]

$$a_1 = -\frac{1}{k} \ln A, \qquad b = \frac{1}{k},$$

we can write

$$t = a_1 + b \ln x. \tag{3}$$

Note that the constant, a_1, replaces a constant of integration, whereas the constant, b, is derived from the velocity constant of the process of growth (or decay). Whereas the former constant will depend on the time when the growth (or decay) started and so will change arbitrarily, the latter will not change so long as the chemical or mechanical conditions of the process remain unchanged.[10]

[8] This is equivalent to the statement given above that the number of solute ions attracted by one surface ion of the crystal is the same for each surface ion (assuming isotropy).

[9] The subscript 1 was added to the constant, a, for reasons which will become clear later.

[10] This important relationship, $1/b = k$, connects a fundamental statistical constant of the distribution of particle sizes with the velocity constant of their growth.

Equation 3 is obviously a transformation, hereinafter called the "logarithmic transformation," which transforms sizes, x, into times, t. Each individual particle will of course begin to grow at an individually different *random* time. The times, t, will therefore be distributed according to some statistical law. By the logarithmic transformation (3) we have transformed the problem of distribution of sizes into a problem of distribution of times. Which distribution of times shall we now assume?

To answer this question, we should note an essential difference between the scale of x and the scale of t. The scale of the times has an *arbitrary* zero and may therefore assume negative as well as positive values. (Cf. our numbering of the years B.C. and A.D.) The scale of x has no such arbitrary zero because no size can be smaller than zero. The distribution of the times is therefore free from the restriction encountered in the distribution of the sizes. There is therefore no objection to assuming the *distribution of the times* to be *normal*. In so doing, we extend, of course, the range of the times from minus infinity to plus infinity. This is permissible since we may assign *asymptotically*, $t = -\infty$, to the initial state of the crystallization when it starts with the nuclei present at the beginning. Also, we may assign the asymptotic value, $t = +\infty$, to the end of the crystallization process.

However, it should be borne in mind that this is only a convenient assumption for the distribution of the times during which the individual particles have been growing. We may as well make a different assumption if we have special reasons to adopt one. Lacking these, the normal distribution is one to which, in the present case, there is no serious objection. Moreover, it is usually regarded as the best expression for what is called "randomness." Obviously, there is a good reason to assume that the times of growing of the individual particles are distributed at random.

The distribution of x which follows from the assumed normal distribution of the times by means of the logarithmic transformation (Eq. 3) is called the *Logarithmico-Normal*, or the *Log-Normal Distribution*.

In Fig. 2 is represented the transformation of a Normal Frequency Curve (broken line) into a Log-Normal Frequency Curve (full line), according to Eq. 3. On the axis of abscissae, we plot the values of t and x, respectively, which correspond to each other by this equation. At the abscissa, t, we have plotted the ordinate,

$$z = \exp(-t^2/2)/\sqrt{2\pi}, \tag{4}$$

of the Normal Frequency Curve. This frequency curve is called *standardized* because its standard deviation is one and its mean is zero. The area below it is equal to unity.[11] The abscissa, t, of such a stand-

[11] Actually, the curve drawn in Fig. 2, which is taken from observations on the emulsion, Seed Lantern 6102 (to be discussed in a later paper), has an area equal to 1461 instead of 1, where $N = 1461$ is the number of grains counted and classified by the observer.

ardized curve is called the *Normal Deviate*. In equating t to a Normal Deviate we have assumed that the mean value of the times is zero and the standard deviation around it is unity. This is permissible since time-zero and time-unit are arbitrary. The value of z corresponding to a given t must be taken from a table of the probability functions (12). The same ordinate is plotted at the corresponding abscissa, x. We so obtain points of the Log-Normal Frequency Curve, for example, A′ corresponding to A, or B′ corresponding to B, etc.

The Log-Normal Curve, it will be seen, is limited at the left by zero, which is as it should be. (The logarithm of a negative value of x is not real, but complex.) The curve is, moreover, skewed, that is to say, asymmetric with respect to its *mode*, which is the abscissa of the peak.

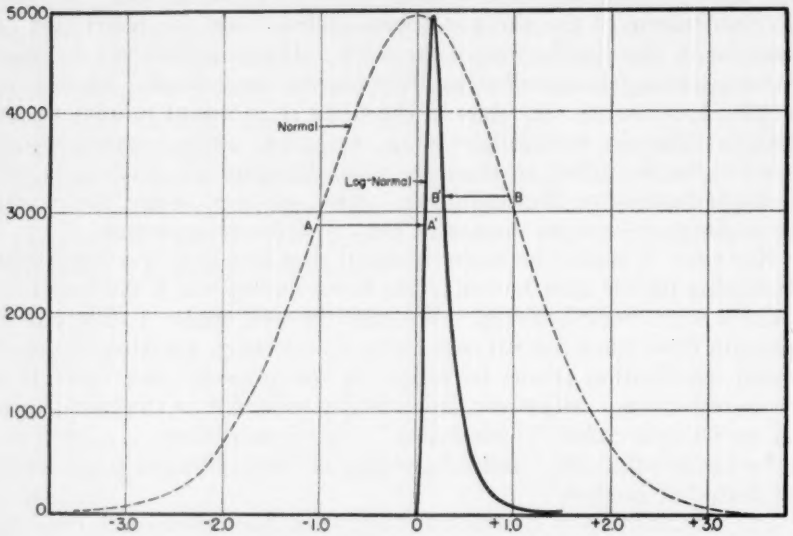

FIG. 2. Transformation of a normal frequency curve (A, B, . . .) into a log-normal frequency curve (A′, B′, . . .). (Seed Lantern No. 6102.)

For sizes smaller than the *modal* size (that is, the most frequent size), there is only a finite spread from zero to the mode, whereas the sizes larger than the modal size cover an infinite spread, that is, from the mode to $+\infty$. This is as it should be. Any curve which is to represent a size distribution will not be symmetric, in general, because it is limited at the left by zero and unlimited at the right. In special cases, we might have a size distribution which is limited on both sides, that is to say, covers a *finite* range (see below). But even in such cases asymmetry is the rule and symmetry is the exception. The latter would have to be explained by a special assumption about the conditions of growth. From this it is seen that skewness is the rule with size dis-

tributions, and this constitutes another argument against the application of the Normal Law to such problems. This is in addition to the argument previously mentioned concerning the negative sizes being implied in the Normal Distribution.

Instead of the frequency curve, one often uses the *cumulative curve* (also called ogive or distribution curve, or probability curve). Such a curve describes how the area under the frequency curve increases as the abscissa increases, and has the shape of the letter "S" (sigmoid curve). It is customary to express the area as a fraction of the total area, so that the ordinates increase steadily from zero to unity or 100 per cent. The abscissa of the ordinate, 0.5 or 50 per cent, is called the *median*. At the

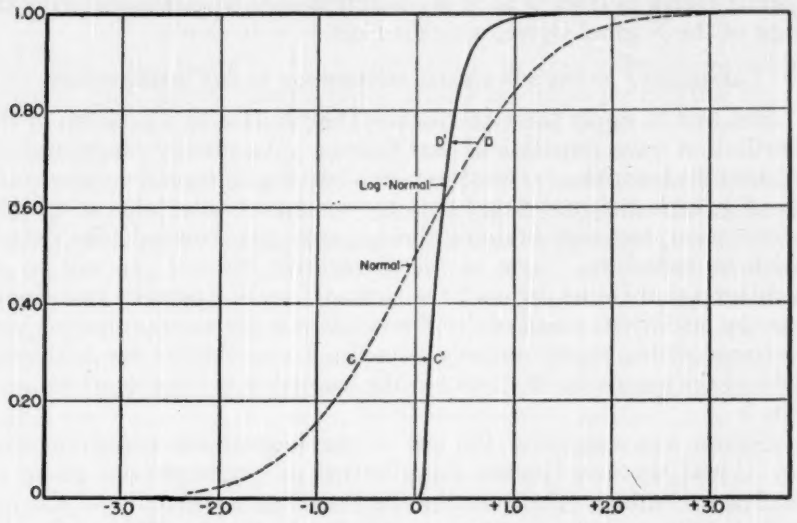

FIG. 3. Transformation of a normal ogive (C, D, . . .) into a log-normal ogive (C', D', . . .). (Seed Lantern No. 6102).

median, one half of the area of the frequency curve is at the left and one half is at the right of it. If the frequency curve is symmetric, as is the case with the Normal Curve, the median coincides with the mode. For the Log-Normal Curve, the median is at the right of the mode.

The Normal Ogive can be written

$$P = \int_{-\infty}^{t} z\, dt, \tag{5}$$

where P is called the Probability and z has the value given by Eq. 4. According to Eq. 5, the median is given by $t = 0$ (since $\int_{-\infty}^{+\infty} z\, dt = 1$).

Again we have a transformation of a Normal Ogive into a Log-Normal Ogive, which is similar to that given in Eq. 3,

$$t = a + b \ln x. \tag{3a}$$

The only difference is that the constant, a, is not the same as the constant, a_1, in Eq. 3, because of the fact that the median of the Log-Normal Distribution does not coincide with its mode.[12]

In Fig. 3[13] we plot again the values of t and x, respectively, on the axis of abscissae and assign to both the same ordinate P as given by Eq. 5. We so obtain the Log Normal Ogive (full line) from the Normal Ogive (broken line), corresponding points being, for example, C and C′, D and D′, etc.

The Log-Normal Ogive is asymmetric with respect to the median, its average slope being steeper for abscissae smaller than the median than it is for abscissae larger than the median. The range of the Log-Normal Ogive is from 0 to $+\infty$, which should be compared with the range of the Normal Ogive, which is from $-\infty$ to $+\infty$.

APPLICATION OF THE LOG-NORMAL DISTRIBUTION TO SIZE DISTRIBUTIONS

The first to apply the Log-Normal Distribution to a problem of size distribution (of a population) was Galton. As already mentioned, he deduced the logarithmic transformation (see Eq. 3) from the exponential law of growth and postulated that the most probable value is the geometric mean, thereby obtaining the Logarithmico-Normal Distribution which he called the "Law of the Geometric Mean." It will be remembered that Gauss deduced the Normal Law of Errors by postulating that the arithmetic mean of the observations is the most probable value. By transforming logarithmically as in Eq. 3, one obtains the arithmetic mean of the logarithm of x which is the logarithm of the geometric mean of x.

Galton also suggested the use of the logarithmic transformation, Eq. 3, and the Log-Normal Distribution in psychophysics since, according to Fechner, the sensation increases proportionally to the logarithm of the stimulus and not to the stimulus itself. This application of the Log-Normal Distribution will be discussed elsewhere.

The first to suggest the application of the Log-Normal Distribution to the size distribution of silver halide grains was R. P. Loveland, in 1924.[14]

Another author to apply the Log-Normal Distribution for particle sizes was Drinker in 1925 (13), who used it for the analysis of dust particles. He followed the experimental method of Green.

The same distribution was applied to powders of silica, granite, and calcite by Hatch and Choate in 1929 (14). The Log-Normal Distribution was also discussed by Austin in 1939 (15), who showed that Mar-

[12] Note that $t = 0$ corresponds to the mode of the Log-Normal Distribution in Eq. 3 and to its median in Eq. 3a.

[13] Figure 3 is also taken from the observations on the emulsion, Seed Lantern 6102 (to be discussed in a later paper).

[14] See the quotation by Sheppard *et al.* (8) p. 529.

tin's measurements of crushed quartz (6) were well fitted by it. He also applied the distribution to measurements by different investigators on clay, alkali carbonates, powdered alumina, etc.

APPLICATION OF THE LOG-NORMAL DISTRIBUTION TO BREAKAGE OF SOLIDS

The Log-Normal Distribution has been applied (see, for example, the above-quoted paper of Austin) to sizes of particles obtained by breakage from large fragments of matter. The question arises whether this application is legitimate from the point of view just given. Obviously, the exponential law of growth must be replaced by the exponential law of decay,

$$\frac{dx}{dt} = -kx. \tag{1a}$$

That such a law holds for the comminution process of breakage can be deduced from simple assumptions similar to those underlying Eq. 1. We have only to assume that the relative rate of decay in breakage is independent of the absolute quantity of material present.

Precisely the same assumption is made in several recent papers by Epstein (16) in order to explain "that the particle size distributions obtained from some breakage processes appear to be logarithmico-normal." His assumption (*b*) is that "the distribution of pieces obtained from the application of a single breakage event to a given piece is independent of the dimensions of the piece broken." This is equivalent to our assumption as formulated in the preceding paragraph. However, Epstein goes beyond the argument as presented above and *deduces* that the distribution of the logarithm of x must be asymptotically normal, whereas we have here *assumed* it to be normal. For that purpose, he avails himself of the Central Limit Theorem of mathematical statistics (17). His argument can be somewhat less rigorously rendered as follows: Suppose that the breakage process is the product of many independent elementary processes. Each such process can be regarded as a random event. The product will therefore be a product composed of many random factors. Its logarithm will therefore be the sum of many random variables. Such a sum, provided only that the number of these variables be increased indefinitely, is normally distributed according to the Central Limit Theorem. In other words, if we increase the number of elementary steps indefinitely they will add up to something which is normally distributed.[15]

Applying this method of reasoning to the process of growth, we may consider it proceeding by steps, a step corresponding, for instance, to the attachment of a single ion. Let the interval between consecutive steps be Δt. Each Δt can be regarded as small and independent of the

[15] A popular version of the Central Limit Theorem is: If an observation depends upon the sum of the effects of a large number of small causes acting at random and independently of one another, the distribution of the sum tends to be Normal.

others. The sum of the Δt's adds up to the time of growth, t, of the crystal as a whole. This time will therefore be normally distributed if the number of steps increases indefinitely.

In concluding, we mention that other laws have been suggested for the analysis of the sizes of broken particles. We cite Gaudin (18) and Roller (19). Both authors have used laws with a "truncated" range. The large sizes are excluded; these laws apply only to small sizes and fail if the sizes become larger. For the reason already given in discussing the truncated Normal Law, we need not discuss these laws further.

Another law was proposed and applied to broken coal by Rosin and Rammler (20). It is supposed to cover the entire range. However, it fails badly in the vicinity of the size zero, where it may lead to an infinite class-frequency. It will not be considered here.

Summing up, we find that the Log-Normal Distribution applies to all particles obtained by crystallization or by comminution, provided these processes satisfy differential Eq. 1 or 1*a*.

KAPTEYN'S "NORMALIZING" TRANSFORMATION

Kapteyn (21) and Kapteyn and Van Uven (21) have given another derivation of the Log-Normal Distribution from a general principle of "normalizing" various statistical distributions. It has been adopted by several authors, Sheppard and his collaborators, Loveland and Trivelli (22, 23) and Gaddum (24). We shall discuss it briefly and explain why we do not accept it here but follow the older derivation of the Log-Normal Distribution given by Galton.

Kapteyn's argument can be summed up as follows: Basically, any distribution dealing with phenomena of growth and the like is assumed to be Normal. If a distribution is found to be not normal, this is attributed to the fact that the independent variable, x, was not chosen appropriately. Instead of measuring x, one should measure some function of x, say $T(x)$. Transformed to this new variable, T, the distribution should be Normal, that is to say, $T(x)$ is a "normalizing" transformation for x.

This has been severely criticized by the eminent statistician, Karl Pearson, who called it "the creation of a shadow variable, which leaves one no further advanced than the original." This criticism refers to the lack of meaning of the new variable, T, in the general case of normalizing. In the special case of the Log-Normal Distribution, the variable, T, is the time of growth, t, as we have seen in Eq. 3, and has therefore a physical meaning. To this criticism we shall add two further arguments: First, Kapteyn and Van Uven have to assume different laws of growth according to the different forms of their normalizing function, $T(x)$. However, there is no need for such laws since, as we shall see, the Exponential Law of Growth is well established in the field with which we are dealing. It is, therefore, best to derive the Log-Normal

Distribution directly from the Exponential Law of Growth, as we did before. It should be noted that the essential point in this derivation is the logarithmic transformation, Eq. 3, and not the assumption of a Normal Distribution of the times, t, which could be replaced by some other distribution of the times, as explained in a previous section. By this derivation we also avoid the assumption of Sheppard and his collaborators, that the initial distribution of the grain sizes is Normal and only changes to Log-Normal under various conditions of growth. We shall, on the contrary, assume that the growth follows the Exponential Law from the beginning and is always Log-Normally distributed.

The other argument against a general principle of normalizing is given by the following mathematical reasoning: Normalizing transformations are frequently used in mathematical statistics for transforming the distribution curve (ogive) into a normal curve. Here we need both curves, the cumulative distribution curve as well as the non-cumulative frequency curve, since both curves are required for size-frequency analysis. Obviously, the Normalizing transformation, $T(x)$, must transform both curves simultaneously into Normal curves. It can be shown that this is only possible if the transformation is the logarithmic transformation given by Eq. 3. [See Appendix I.]

In what follows we shall, therefore, regard any confirmation of the Log-Normal Distribution of particle sizes as a confirmation of the Exponential Law of Growth. Conversely, if we encounter any disagreement, we shall have to regard it as a failure of that law.

THE LAW OF LIMITED GROWTH

Here we must insert an important remark about the validity of the Exponential Law of Growth.

Consider Eq. 2. This equation implies that the growth starts from zero when $t = -\infty$ and ends with infinite size when $t = +\infty$. This is obviously an absurdity. There cannot be a size zero nor can there be an infinite size.

Moreover, the rate of growth increases indefinitely according to Eq. 1. This is contrary to common experience. Most phenomena of normal growth have a rate which increases only during an initial period and afterwards decreases gradually.

This form of the Law of Growth cannot have general validity. It is now obvious why it has no counterpart in chemical reaction-kinetics, as mentioned before. Let us consider whether we can amend these defects so as to obtain a law of growth of general validity.

Obviously, we must introduce a lower limit (say), x_0, which must be >0 and an upper limit (say), x_∞, which must be $< \infty$.

In our problem, the lower limit will be the size of the *nuclei* which have to be present before crystallization can start. This size will be small and may be even submicroscopic. It will, however, be >0, since

the smallest crystal which can exist is the so-called unit cell which consists of several atoms. It is true that, for the most part, such a size, x_0, will be negligible compared to the sizes, x, which occur in microscopic work. However, let us, for the moment, introduce the limit, x_0, in our formulae.

All that is necessary is to write the differential equation in the form,

$$\frac{dx}{dt} = k(x - x_0), \tag{1b}$$

which possesses the integral,

$$x - x_0 = A \exp kt. \tag{2b}$$

These formulae simply replace Eqs. 1 and 2. This does not involve a major change in the conclusions drawn from these equations. We need only replace the sizes, x, by the size differences, $x - x_0$. The logarithmic transformation is now

$$t = a + b \ln (x - x_0). \tag{3b}$$

This leads to a Log-Normal Distribution of the differences, $x - x_0$. If we plot the frequencies *versus* the sizes, x, or the size differences, $x - x_0$, on ordinary charts, such as in Fig. 2 or 3, we obtain, in either case, curves of the same shape. The only difference is that in the first case the curves start not at the origin but somewhat to its right. It is different if we use a logarithmic scale as in the so-called Log-Probability Charts (see Figs. 4–8, Part II). If we plot on these charts against the size differences, $x - x_0$, we get a simple type of curve, that is, straight lines. However, if we plot against the sizes, x, we have a change of shape, a curvature at the lower end where x is equal to or a little larger than the critical value, x_0.

It is obvious that a Log-Normal Distribution of the type (3*b*) contains three parameters, a, b, x_0, instead of only two parameters for the simple distribution of type (3). These values, a, b, x_0, have to be estimated in any practical case from the entire data in the same way as the values a, b have to be determined in the simple case (3) (see a later paper).

As already mentioned, there is no need to use the third parameter in the present problem, where we may safely regard x_0 as approximately equal to zero. In other cases, a lower limit, $x_0 > 0$, has been used by some authors. See, for instance, a paper by Sheppard and collaborators (25) where photographic fog was considered. (See also Loveland and Trivelli, ref. 22.) It has also been used by Kapteyn (21) and in biology, by Gaddum (ref. 24, p. 465).

We now consider an upper limit, x_∞, which is finite. This induces a major change in our formulae.

Instead of Eq. 1 we write now

$$\frac{dx}{dt} = k\,\frac{(x - x_0)(x_\infty - x)}{x_\infty - x_0}, \tag{1c}$$

which takes care of the upper limit as well as of the lower limit. (The latter could, of course, be set equal to zero.) The integral is now

$$\frac{x - x_0}{x_\infty - x} = A \exp kt. \tag{2c}$$

It will be seen that x is now contained between the limits, x_0 and x_∞, the former corresponding to $t = -\infty$ and the latter to $t = +\infty$.

Moreover, the law (1c) accounts for the aforementioned behavior of the rate of growth. This rate increases, according to Eq. 1c, in the initial period, approaches a maximum at an inflection point in the middle of the range, and decreases thereafter.

Mathematically speaking, Eq. 1c is similar to the equation of the second-order or bimolecular reaction in chemical kinetics. An example is the case where two kinds of molecules are produced at the cost of one or two other kinds of molecules. The chief difference seems to be that the rate of such a chemical reaction behaves as a monotone decreasing function. In other words, there is no inflection point inside the chemically significant interval. Moreover, there is also a difference in the definition of the constant of velocity. It is here assumed to be of the same dimension, $(\text{time})^{-1}$, as for the first-order Eq. 1. In physical chemistry it is — unfortunately — different.

The transformation is now

$$t = a + b \ln \frac{(x - x_0)(x_\infty - x_0)}{x_\infty - x}, \tag{3c}$$

and this yields a distribution differing from the ordinary Log-Normal Distribution given by Eq. 3. It has four parameters, a, b, x_0, x_∞. We shall call it the *Limited Log-Normal Distribution*[16] to distinguish it from the ordinary or *Unlimited Log-Normal Distribution*. It transforms, obviously, into the latter if we introduce a new variable,

$$X = \frac{x - x_0}{x_\infty - x}(x_\infty - x_0).$$

The law, given by Eq. 2c, will likewise be called the *Exponential Law of Limited Growth*, whereas the law given by Eq. 2 will be called the *Law of Unlimited Growth*.

It is easily seen that for $x_\infty \to \infty$ and $x_0 \to 0$ our new formulae revert to the old formulae and our present law to the Law of Unlimited Growth.

[16] Such a case was considered in unpublished work by Loveland and Trivelli, and Sheppard (26), where these authors refer to an earlier suggestion made by Van Uven. Compare also the logistic law of Verhulst.

Ordinarily, in microscopic work, we can safely regard x_∞ as infinite and x_0 as zero. There is, however, at the upper end of the range, some trace of the existence of a finite upper limit such as any particle distribution obviously possesses even though that limit be macroscopic. We shall occasionally have to point out this fact.

The distribution (3*c*) has been tested with other phenomena which exhibit a more marked upper limit than microscopic particles do. This will be reported elsewhere.

(To be continued.)

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

OPENING ACTIVE AMPOULES BY REMOTE CONTROL

A new oven loading technique, developed by Jesse Sherwood of the National Bureau of Standards for use in the atomic-beam clock program, overcomes one difficult problem of atomic beam experimentation —the introduction of highly reactive materials, such as alkali metals, into the "oven" which acts as the beam source. In the past the procedure has been to place an ampoule of the material to be studied into the oven, crush or otherwise open the ampoule, and set the oven into position as quickly as possible. The system was then evacuated before much reaction with the atmosphere could take place; consequently the success of the operation depended on having everything in readiness for the "run."

In the new technique developed by the Bureau's Microwave Standards Section, the difference in the thermal expansivity of aluminum and glass is utilized. The oven used with the atomic-beam magnetic resonance apparatus is of a conventional type, but the cesium metal—the frequency stabilizing element of the new atomic beam clock—is distilled into a specially designed ampoule and sealed off under vacuum. The ampoule contains a well into which is inserted a carefully lapped aluminum plug of about 7-mm. diameter. This plug is removed before filling and then set firmly in place after sealing. When the ampoule is heated to 80–100° C., it will crack because of the larger thermal expansivity of aluminum, referred to glass.

With this method, it is possible to load the oven, carry out further checks on the experimental conditions, and pump out the system before exposing the active metal. There is thus less chance of clogging the very narrow (0.001 in.) slits used to collimate the beam of cesium atoms, and the entire apparatus may be made vacuum tight and otherwise operative before a run is attempted.

Solutions of these problems are quite important in an apparatus designed for long-term operation as a frequency and time standard of unprecedented accuracy; calculations show that utilization of the quantum transitions of a cesium atom beam in the new atomic beam clock will provide an ultimate accuracy of 1 part in 10 billion. The self-opening ampoule represents a solution to only one of many problems and has the very desirable feature of simplicity, a virtue not always obtainable in atomic beam work.

* Communicated by the Director.

ISOTOPE EFFECT IN SUPERCONDUCTIVITY

Recent low-temperature experiments conducted by Dr. Emanuel Maxwell of the National Bureau of Standards have resulted in the discovery of a new and wholly unexpected relationship between superconductivity—the loss of electrical resistance at very low temperatures—and the constitution of the atomic nucleus. It was found that the pure mercury isotope 198 becomes superconducting at a temperature about 0.02° K higher than does natural mercury, which is a mixture of five different isotopes having an average atomic weight of 200.6.[1] As the difference between the isotopes depends entirely on their nuclear masses, this shift, which is proportionately quite large for a temperature so near absolute zero, indicates that the nucleus must exert an important effect on the superconducting properties of the metal.

Until now it has been generally supposed that superconductivity is concerned exclusively with the properties of the electronic configuration outside of the nucleus. However, the results obtained at the Bureau, in combination with an independent and almost simultaneous discovery of the same phenomenon in other mercury isotopes at Rutgers University,[2] establish the validity of the nuclear effect beyond question. The two researches are complementary and when taken together provide a conclusive demonstration of the effect.

The phenomenon of superconductivity, characterized by the complete disappearance of the electrical resistance of metals such as lead and tin at liquid-helium temperatures, was discovered in 1911 by the Dutch physicist, H. Kammerlingh Onnes. Soon after this discovery, it was found that resistance reappears when a large electric current is sent through a superconductor or when a sufficiently strong magnetic field is applied. In 1918, Dr. F. B. Silsbee of the National Bureau of Standards postulated[3] that resistance is restored when the magnetic field reaches a value known as the *critical field* regardless of whether the field is applied externally or is caused by the current in the conductor. This theory, known as the Silsbee hypothesis, has been verified for pure metals in numerous experiments.

Although a great deal of research has been done in the last few years on the remarkable properties of matter at low temperatures, there is as yet no adequate theoretical explanation of superconductivity. To provide a basis for a better understanding of the properties of matter, the National Bureau of Standards is now making studies seeking a more complete explanation of this and other low-temperature phenomena. In the course of the studies, it was suggested that the *transition temperatures* at which different isotopes of the same element become super-

[1] E. MAXWELL, *Phys. Rev.*, Vol. 78, p. 477 (1950).

[2] C. A. REYNOLDS, B. SERIN, W. H. WRIGHT, L. B. NESBITT, *Ibid.*, Vol. 78, p. 487 (1950).

[3] FRANCIS B. SILSBEE, "Note on electrical conduction in metals at low temperatures," *Bull. B. S.*, Vol. 14, p. 301 (1918) S307.

conducting be investigated to determine whether the mass of the atomic nucleus has any effect on this temperature.[4]

Actually, the transition temperatures of the mercury samples were not measured directly. Instead, the critical field was measured as a function of temperature in the neighborhood of the normal, or zero-field, transition temperature. Then, by extrapolating to zero field, the normal transition temperature was accurately determined.

In measuring the critical field, use was made of the Meissner effect, or zero induction property. This is the name given to the property of a superconductor which prevents a magnetic field from existing within it. Thus, if a superconductor is brought into a magnetic field, the field cannot penetrate to the interior. Alternatively, if the metal becomes superconducting while immersed in a magnetic field, the field is immediately expelled. In other words, a superconductor acts like a perfectly diamagnetic material.

In the Bureau's experiment, the superconductor, a long thin needle of mercury enclosed within a sealed capillary tube, was placed inside a pick-up coil consisting of many turns of fine wire. The superconductor and pick-up coil were immersed in liquid helium within a Dewar flask surrounded by liquid air, and the pick-up coil was connected to a ballistic galvanometer. The critical field for a given temperature was then obtained by slowly increasing the current in a set of Helmholtz coils surrounding the liquid air bath. When the critical field was reached, the specimen suddenly lost its superconductivity and was penetrated by the magnetic field of the Helmholtz coils. The resulting change in flux linkages in the pick-up coil induced a voltage which caused a deflection of the galvanometer.

By varying the pressure within the dewar of liquid helium, the temperature of the specimen was controlled in the range between 1.5° and 4.2° K with relative ease. The experiment was repeated at a number of different temperatures, and the critical field was determined for each temperature. The critical field was then plotted against temperature, and the resulting curve was used to determine the transition temperature of the material under study. In this way it was found that pure Hg^{198} has a zero-field transition temperature of 4.177° K as compared with 4.156° K for natural mercury.

To make sure that the effect observed was due to the difference in mass and not to chemical impurities present, the mercury samples were carefully distilled to rid them of impurities. As the mercury had originally been prepared by bombarding gold with neutrons, there was some possibility of residual gold contamination. However, it was not

[4] Earlier experiments by Kammerlingh Onnes (1925) and Justi (1941) on the transition temperature of uranium lead (a lead derived from uranium by natural radioactive decay processes; atomic weight, 206.0) as compared with that of natural lead (atomic weight, 207.2) had indicated no deviations within experimental error.

possible to make a spectrochemical analysis of the Hg^{198}; instead 0.1 per cent of gold was added to some natural mercury, and the critical field curve of this material was determined. Within experimental error there was no difference between the curve for pure natural mercury and that for mercury with gold added. It was therefore concluded that gold contamination, if present in the Hg^{198}, was unimportant.

In the work at Rutgers University, samples of mercury which had been enriched electromagnetically in each of the three isotopes Hg^{199}, Hg^{202}, and Hg^{204}, respectively, were studied. Although these samples did not have the high degree of isotopic separation of those used at the Bureau, they provided a greater number of individual isotopes for comparison with natural mercury. When the Bureau's results are combined with those of Rutgers in a plot of transition temperature against average atomic mass, a smooth curve is obtained which shows a definite relation between the temperature at which mercury becomes superconducting and the mass of its atoms.

Since the initial discovery of the isotope effect in the superconductivity of mercury, a sample of tin consisting mainly of Sn^{124} has been investigated at the Bureau and found to exhibit a similar shift in transition temperature. The sample contained other tin isotopes as impurities so that its average atomic mass was 123.1. The transition temperature determined at the Bureau was 3.662° K, as compared with a value of 3.715° for natural tin, whose atomic mass is 118.7.

As yet the true significance of the relationship between nuclear mass and superconducting transition temperature is not clearly understood. It has been suggested that the shift in transition temperature with nuclear mass may be associated with a corresponding change in atomic spacing. As it has already been found that an increase in spacing due to stress results in a higher transition temperature, this suggestion offers interesting possibilities which are now being explored at the Bureau.

SLIDE RULE FOR COMPUTING RADIOACTIVITY DECAY FACTORS

The scientist or technician concerned with the measurement of radioactivity is always confronted with the necessity of knowing the strength of a source at some particular time other than that at which it was last measured. This requires knowledge of the half-value period, or the decay constant, from which the decay correction-factor can be computed from the conventional formula. To facilitate this procedure, J. L. Herson of the Radioactivity Laboratory at the National Bureau of Standards has devised a special slide rule as an effective time saver when many computations of decay factors are involved.

The slide rule includes two scales laid off side by side and properly chosen to provide a graphic computation of the relationship expressed

in the usual decay-correction formula. The bottom scale, L, consists simply of a uniform scale of arbitrary length graduated from 0 to 1, while the upper scale, M, is derived from an intermediate scale in the following manner. The entire length—same as scale L—is taken to be 0.301 (log 2) with the left and right ends designated 1 and 2, respectively, Sub-divisions are located at distances from the left corresponding to the logarithms of the respective numbers. This scale, inverted end for end and the numbers divided by 2, becomes the M scale.

In use, the slider is moved to the division on the scale L corresponding to the fraction of half life, that is, t/t_h where t is the elapsed time and t_h is the half life of the activity. The remaining fraction of the activity is then read on scale M.

Scale L will be recognized as the usual uniform scale on the conventional slide rule. Therefore by adding an M scale to the standard slide rule, computations of the decay factor can be made in two simple steps using only the numbers t and t_h. The reduction in the number of steps and the effective spread of the scale expedites the computation and provides greater accuracy in determining radioactivity decay factors.

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MUSEUM

A great ship is a thing of beauty and one of the noblest products of man's mind and hands. Its vast engines must develop 100,000 horse power, yet not a panel in the dining room may rattle. The marvel of a ship grips one's imagination as she rides like a cathedral of steel on the water, but greatest of all her marvels is the surety with which she travels over the trackless wastes of the ocean.

The bridge of a ship is a place of enchantment to the landsman. The crude instruments of the early mariners have been replaced by precision instruments which eliminate much of the labor and some of the errors attendant upon human handling. So much of what was the fruit of observation and calculation is now rendered by automatic instruments.

Through the courtesy of the United States Department of the Navy, the visitors to the Institute's museum are able to see and to handle the instruments which are to be found in the control room and on the bridge of a small cruiser or destroyer. Probably the most interesting item of the equipment is the automatic steering device, designed to remove routine drudgery and possible error from the navigator's constant vigilance. The integral part of this is the gyro-compass which, although shown here, in reality lies snug and secret in a little square room between decks. Its function is to rotate in one plane and to register an instant protest to any change in the plane of its rotation. The "mother" instrument communicates with "daughter" instruments located at strategic points on the ship, where readings can be made on the ship's course.

The gyro-compass is connected with the automatic steering device so that, once the course has been laid, the compass exercises an enigmatical and vigorous control over the ship's direction. Let wave or wind or current cause the ship's bow to deviate from the set course and the automatic steering device immediately makes a correction under the influence of the compass.

Many traditions and legends of the sea surround the old custom of heaving the lead to ascertain the depth of water under a vessel, but this task is now surrendered to an instrument which not only instantaneously sounds the depth of the water but also keeps an automatic record of its readings.

Another function of the seaman which formerly was picturesque enough but subject to erratic results was the use of the "log," a device by which the speed of the ship was reckoned. Many ingenious methods were adopted in the past but those in use today leave nothing to chance since they are based upon proved scientific principles, involving the measurement of water pressure or the revolutions of the propeller.

These are but a few of the many devices which are shown in the museum as requisite for the safe control of a ship. Others demonstrate the complicated system of communications which a vessel must carry to ensure a prompt response to orders and to permit the exercise of authority from a single point.

Navigation from its earliest days has been founded upon the scientific knowledge of the times. It is inevitable, then, that scientific development should be reflected in the instruments placed in the navigator's hands. This is shown convincingly in the assembly of instruments on a modern ship's bridge. The fruits of many sciences are required to contribute to the equipment of the modern ship, and the quickness of those in authority to seize upon new developments and to turn them to good account is shown in this equipment. Practical science is liberally applied, and the most advanced technology finds its applications in the perfection of construction. A story could be written upon every one of these pieces of equipment and most of those stories would show that man has been struggling without cessation to conquer the peril of the ocean, to simplify the arduous life of the seaman, and to improve the safety of those who, for pleasure or for gain, have to travel by water. This struggle is well illustrated in the collection of instruments in the Marine Transportation Section, where the progress in the aids to navigation is shown. A visit to the ship's bridge will convince anyone how efficient is the mechanical control of the little world encompassed in every ship, and how readily information upon conditions can be gathered and steps taken to meet emergencies.

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THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

***Abstract of* Control Mechanisms for Hydraulic Piston Pumps.**[1]—W. B. STRICKLAND[2]. Hydraulic applications have increased with improved flow controls. The most effective method of controlling the fluid is at the pump. The designs of both the radial and axial piston pumps lend themselves to variable-delivery by simply changing the pump displacement. Thus, the output can be altered without changing the speed of the motor. This method eliminates the power loss that is caused by either throttling or bypassing fluid. In addition, such a control device can change the direction of flow without requiring special valves or altering the *direction* of the driving motor. With these controls, the pump is well adapted for both heavy-duty service or for accurate drive control of such equipment as machine tools. The following sketches illustrate the method by which the pump displacement and pressure are regulated:

RADIAL PUMPS

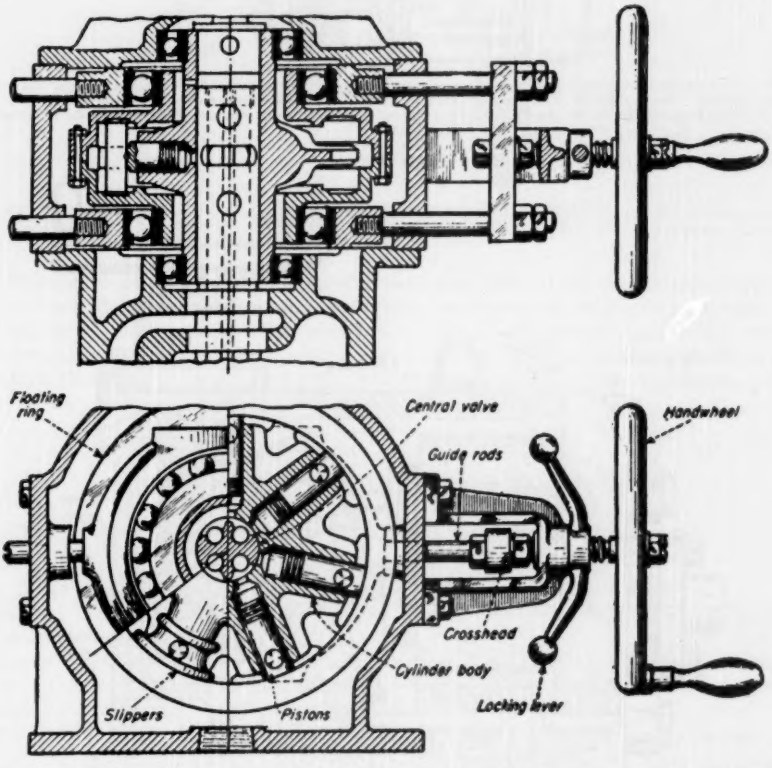

FIG. 1—Manual displacement control by handwheel screw moves floating ring to an off-center position. Cylinder body is driven on-center and any eccentricity of the floating ring causes relative motion between the pistons and cylinders during each revolution. Adjusting floating ring changes flow from zero to maximum in either direction. This type is used where high pressures are required.

[1] Data by courtesy of American Engineering Company, Philadelphia, Pa. Published in *Product Engineering*, August, 1950, pp. 120–121.

[2] Research Engineer (Hydraulics), The Franklin Institute Laboratories for Research and Development, Philadelphia 3, Pa.

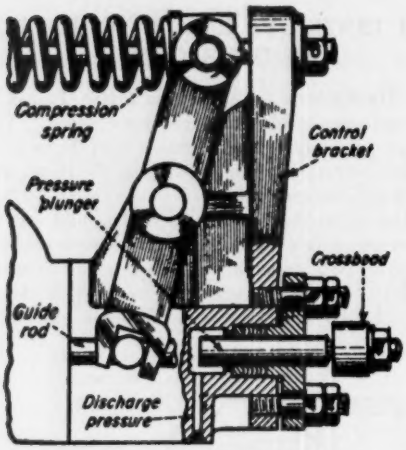

FIG. 2—Automatic pressure regulation is performed by balanced forces acting between plunger and compression spring. Pump remains in full stroke position until pressure on plunger overcomes the spring force and returns pump to a leakage stroke which sustains the pressure. Adjustable nut on spring sets discharge pressure.

AXIAL PUMPS

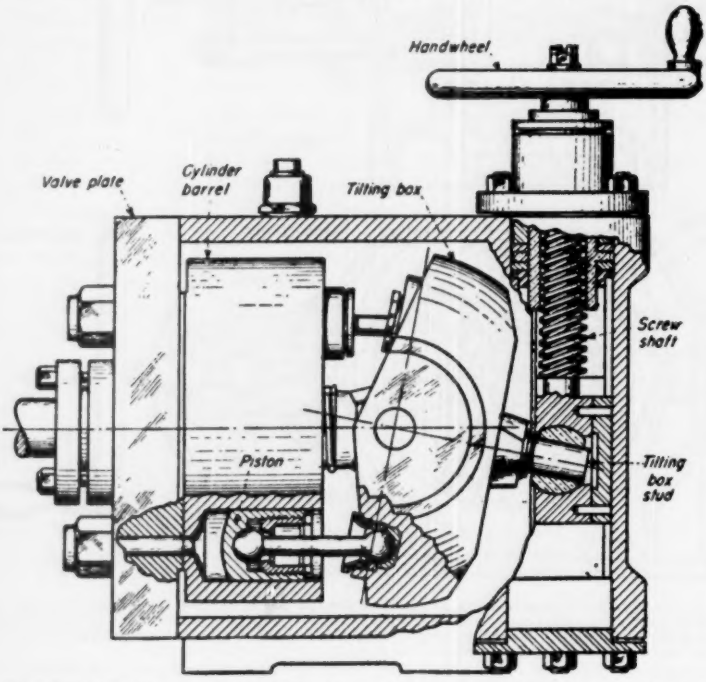

FIG. 3—Manual displacement control consists of handwheel and screw shaft which alters angle of tilting box, or "wobble plate." Larger angle increases the piston stroke and produces higher flow. Reversed flow is possible by simply changing angle to other quadrant. Axial piston pumps are often used where high displacements at lower pressures are necessary.

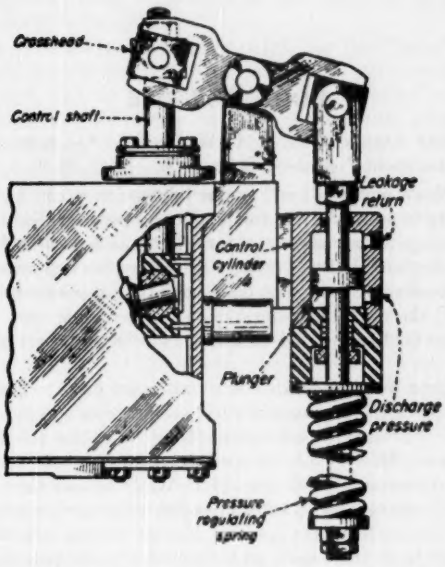

FIG. 4—Discharge pressure is automatically controlled by balanced forces acting between pressure plunger and adjustable compression spring. Pump remains in full stroke position until pressure on plunger overcomes spring force and returns the tilting box stud toward the neutral position.

Control mechanisms for hydraulic pumps have been one of the primary factors in promoting the use of fluid power equipment. Fluid power equipment can accomplish both linear and rotary motion, or any combination of the two. The effectiveness of present day control mechanisms enables fluid power to compete with mechanical or electrical equipment in performance, dependability, weight, adaptability, and price, regardless of the application.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

BLANTON, H. ELMORE: Carrier Compensation for Servomechanisms.

LEE, JAMES S. Y.: Interflection Tables for Rooms with Floors of High Reflectance.

CHARLTON, T. M.: Some Notes on the Analysis of Redundant Systems by Means of the Law of Conservation of Energy.

SILBERSTEIN, LUDWIK: Paucity of Quanta Rendering a Silver Halide Grain Developable.

MOON, PARRY AND DOMINA EBERLE SPENCER: Simplified Interflection Calculations.

EISENBUD, LEONARD: On the Theory of Angular Correlation Phenomena.

GROSS, ERIC T. B. AND LEONARD RABINS: Transient Analysis of Three-Phase Power Systems.

The January 1951 issue will be a special number, commemorating the 125th anniversary of the JOURNAL. In addition to the history of the JOURNAL, there will be fifteen papers by outstanding authorities on the subject of probable future developments in various branches of science and technology. The complete list of authors will appear in the December JOURNAL, together with the titles of their papers. If extra copies of this important issue are desired, write to the Assistant Editor stating the number you wish reserved for you—the cost is one dollar per copy, payable either in advance or on delivery.

BOOK REVIEWS

MOTOR OILS AND ENGINE LUBRICATION, by C. W. Georgi. 514 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp, 1950. Price, $8.50.

Any reference book on motor oils and engine lubrication would warrant attention as the first effort along this line in a decade. In fulfilling its purpose well, this volume merits recommendation. Here is an up-to-date survey on a subject which recently has attained the scope and interest to fully justify a collection of facts. The author has well accomplished his endeavor to collect for handy reference all information on oils and their application to internal combustion engines. The book is, therefore, of unquestionable interest not only to anyone connected with the oil and engine field by way of design, sales or service, but also to the lubrication engineer.

The material covered may be divided into two main parts. The first pertains to the motor oil itself, its properties, accenting viscosity, performance specifications, refining, manufacture and additives. The second part relates the oil to engine lubrication, design, performance and maintenance. Many types of operating difficulty and failure are diagnosed in detail, listing corrective measures and remedies. While automotive applications are considered primarily, diesel, aircraft and stationary engine types are mentioned as well.

The author has accomplished the unusual feat of writing a book sufficiently void of mathematics and theory to be thoroughly understandable to the practical man, yet chock-full of references to interest the technical reader who wishes to follow up any part in detail. The volume contains over 300 references to books, articles and patents, plus some 60 references to ASTM, ASA and federal specifications and procedures.

Subject inclusion is handled very well. Brevity was used to advantage, so that it does not detract in any way from the completeness of coverage. Many examples of typical designs, tests and failures are included to amplify the subject under discussion. Worthy of particular note are the paragraphs evaluating and discussing significance of certain topics, since opinions are not expressed in the standard reference book.

One of the techniques employed which makes the book so useful for both quick reference and study is the repetition of the text material in tables and again in figures. In addition to an index, ample cross-references exist between chapters where an overlap occurs. In all, the volume contains some 160 figures and as many tables and tabulations plus about 20 summary lists for terminology, evaluation, causes and remedies of failures, properties, installation guides and hints on automobile upkeep.

GUNTHER COHN

HARVARD CASE HISTORIES IN EXPERIMENTAL SCIENCE. Illustrations, 15 × 23 cm. Cambridge, Harvard University, 1950.

Case 1. Robert Boyle's Experiments in Pneumatics, edited by James Bryant Conant. 70 pages.

Case 2. The Overthrow of the Phlogiston Theory. The Chemical Revolution of 1775–1789. 59 pages.

Case 3. The Early Development of the Concepts of Temperature and Heat: the Rise and Decline of the Caloric Theory. 106 pages.

These booklets have a significance out of all proportion to their bulk. When Dr. Conant left the home pastures at Harvard to give the twenty-third series of Terry lectures at Yale University he chose as his subject "The Understanding of Science." He made clear at the beginning that he was concerned with the scientific education of the layman and the student who was majoring in the humanities. "Modern science has become so complicated," he said on that occasion, "that today methods of research cannot be studied by looking over the shoulder of the scientists at work. If one could transport a visitor, however, to a laboratory

where significant results were being obtained at an early stage in the history of a particular science, the situation would be far different."

In order to assist the intelligent layman to obtain the "feel" of science, Dr. Conant has now recruited a team of experts who are giving the layman opportunity to visualize the laboratories, the instruments, and the methods of the earlier scientists, and to perceive against the background of knowledge then current just how the scientists achieved those discoveries which are the milestones along the highway of scientific progress. These case histories are admirably designed to show that no great scientific discovery is made overnight. By studying the evolution of a new conceptual scheme we see the difficulties with which the pioneers in science had to contend. The failures, which are always overlooked in the histories of science, are here included, as they must be if the student is to understand the stumbling steps which in actuality constitute progress. The reading of these carefully edited studies will enable a layman to come closer to the scientific mind than any laboratory manual or all the popular scientific magazines.

The individual case histories will undoubtedly vary in interest and in editorial merit, but the first three set a high standard for those which are to follow. They serve as admirable introductions to select phases of physical studies and to see them through the eyes of the men who were conducting the original studies. The case history technique has been adopted in military, legal, and medical history with satisfactory results for the student, and this Harvard essay may very well pave the way for the application of the technique to the study of science.

Science students would be well advised to read these little books. They cannot fail to be a source of inspiration if the student has aspirations to become anything more than a measurer of quantities or the classifier of facts.

T. C.

METALLURGICAL APPLICATIONS OF THE ELECTRON MICROSCOPE. Institute of Metals Monograph and Report Series No. 8. 164 pages, illustrations, plates, 14 × 22 cm. London, The Institute of Metals, 1950. Price, $2.50.

This monograph stems from a symposium held at the Royal Institution in London in November, 1949 under the joint sponsorship of the Institute of Metals and seven other British scientific organizations. In the book are contained 13 papers presented at this meeting along with an account of the discussion which followed them. These papers constitute contributions from leading workers in the field in England, France, Germany, and the United States.

The first paper gives a general account of the development of the electron microscope, the requirements of the metallographer in applying electron microscopy to his work, the necessity for and difficulties involved in employing a replica technique, and a number of metallurgical problems still unsolved which may be fruitfully attacked with the aid of the electron microscope.

Three of the other papers, by authors from France, from Germany, and from the United States, present comprehensive surveys of the techniques used and the metallurgical studies currently being conducted in their respective countries.

The remaining nine papers are devoted to reports of research (in several cases stating reviously unpublished results) upon definite problems of interest in metallurgy. Each author gives detailed descriptions of the techniques he has employed as well as his interpretations of the micrographs obtained. The evaluations of etching reagents and replica materials are particularly thorough and worthwhile.

In view of the wealth of material it contains, this book should have appeal for a wide range of readers—physicists and others with only a casual interest in the subject as well as metallurgists and electron microscopists should find it enlightening. It should be particularly helpful in breaking down the prejudice which some metallographers seem to have retained in the past against the extensive use of the electron microscope as an aid in their research.

Only the lack of an index detracts from the use of this volume as a standard reference for all workers in the field.

JEAN A. MINKIN

PRINCIPLES OF IONIC ORGANIC REACTIONS, by E. R. Alexander. 318 pages, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $5.50.

In recent years there has been a great deal of stress placed on reaction mechanisms in organic chemistry. It has been found that a great many organic reactions undergo the same basic transformations no matter what compounds may be reacting. These basic mechanisms can be represented by electronic configurations, and by this method several atypical as well as normal reactions may be explained.

The contents of this book are divided into two main parts: the first part apprizes the reader of some of the simplest intermediates from which most ionic organic reactions can be built. The second part interprets some of the common reactions such as the Claisen condensation and the Grignard reaction in terms of electronic formulae. The author has attempted to emphasize the similarities between reactions rather than the differences that exist, and to stress the subject of organic chemistry he omits as much mathematics and physical chemistry as possible.

Some chemists still tend to view reaction mechanisms as merely a sophisticated means of representing a reaction, and not as an aid in the understanding of each step of particular transformations with no attempt to replace the functional formulae now in use. The reviewer feels that, if more colleges and universities would incorporate electronic mechanisms in their organic chemistry courses, students would abandon rote learning and be able to understand why a reaction takes place. A few unreasonable mechanisms have appeared for particularly abnormal reactions, but this should not detract from this means of representation.

This book, designated for advanced undergraduates and first year graduate students, is easily read and the organization of material is excellent. The degree of accuracy is very high and this further recommends it as a text book. The author has presented the theories most generally accepted, and has avoided the controversial mechanisms mentioned above. He has included such data as necessary to lend support to the theories without going into undue complexities. It will certainly serve as an excellent text book as well as a valuable addition to the chemist's bookshelf.

DONALD H. RUSSELL

ATOMIC PHYSICS, by Wolfgang Finkelnburg. 498 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., 1950. Price, $6.50.

Atomic Physics, by Dr. Wolfgang Finkelnburg, is a book in which physical phenomena and the structure of matter are explained in terms of the elementary particles and a few fundamental physical laws. Other books in this field have frequently had the term "Modern Physics" used in their titles, but the term "Atomic Physics" is certainly more descriptive since the book deals with nuclear, molecular, and solid state physics from the atomistic point of view.

The book consists of seven chapters. The first chapter is a brief introduction. In the second chapter the author discusses the properties of ions, electrons, nuclei and photons. Some of the pioneer experiments by which many of these properties were first determined and instruments involving their practical applications are described. There is also a brief discussion of the interaction between matter and electromagnetic radiation.

Chapter three explains the origin of atomic spectra. It includes among many other topics discussions of X-ray spectra, the Zeeman and Stark effects, hyperfine structure and the periodic table explained according to atomic theory.

In the next chapter the author discusses the atomic theory in terms of modern quantum mechanics. Here use is made of the Heisenberg uncertainty principle, the Fermi statistics, and the wave-particle dualism of radiation and matter.

The last three chapters deal with nuclear, molecular and solid state physics respectively. A good discussion of the spectra of diatomic molecules is included in the chapter on molecular physics. Although the last chapter is entitled "Atomic Physics of the Liquid and Solid State" only brief mention is made of liquids at the beginning of the chapter. Among the many topics discussed in this chapter are crystal structure, the properties of metals and semi-conductors, ferromagnetism, luminescence, rectification and the photographic process.

This book covers an extremely broad field and is very well written. It is descriptive in nature, involves no difficult mathematics but yet it is not superficial. It is the type of book the physicist or physical chemist will enjoy reading to refresh in his memory many of the fundamentals of atomic physics. It should also prove to be a good text for college seniors and graduate students majoring in physics. Since the author includes no problems or examples, most instructors will probably wish to supplement the course along this line to amplify the principles discussed in the book.

The book is amply illustrated with photographs and drawings. It will be found valuable both as a text and as a general reference book.

M. D. EARLE

ACCEPTANCE SAMPLING; A SYMPOSIUM DELIVERED AT 105TH ANNUAL MEETING OF THE AMERICAN STATISTICAL ASSOCIATION AT CLEVELAND, OHIO, JANUARY 27, 1946. 155 pages, diagrams, 15 × 24 cm. Washington, The American Statistical Association, 1950.

This symposium is a series of papers and discussions on the subject of acceptance sampling. The subject matter is divided roughly into two main headings: (1) Acceptance Sampling by Attributes and (2) Acceptance Sampling by Variables.

The first paper, "Statistical Developments in Acceptance Sampling by Attributes Prior to 1941," by Paul Peach, reviews certain milestones in the history of acceptance sampling. The influence of Shewhart on the developments of the modern quality control chart and that of Dodge and Romig on the development of methods of sampling inspection are stressed.

The second paper, "Wartime Developments in Acceptance Sampling by Attributes," by Edwin G. Olds, reviews some of the major developments during World War II. The application of Dodge-Romig lot tolerance tables are considered and illustrated. Inspection techniques employed by Army Ordnance, Quartermaster Corps, and the Office of Procurement of the Navy are reviewed. Sequential sampling for attributes is considered.

J. H. Curtiss' paper, "Lot Quality Measured by Average of Variability," presents a theory for acceptance sampling by variables, that is, acceptance sampling based on inferences drawn from a sample relating to the frequency distribution of an aggregate. The concepts of acceptance number, power of a test, producer's risk, and consumer's risk are introduced. Formulas are given for sample size and region of rejection both for Gaussian and non-Gaussian distributions of the quality defining variables. The close relation between lot acceptance sampling and process control is emphasized.

The final paper, "Lot Quality Measured by Proportion Defective," by W. Allen Wallis, presents the necessary equations for determining lot acceptance with respect to per cent defective.

The extended discussions which follow the papers clarify many of the points of theory. Many of the definitions and methods are illustrated by simple examples from industry.

This symposium will be extremely useful to personnel engaged in setting up acceptance sampling plans in industries and to students of applied statistics.

HOWARD H. BROWN

INDUSTRIAL INSTRUMENTATION, by Donald P. Eckman. 396 pages, drawings, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $5.00.

As stated by the author, this book is intended primarily for the undergraduate in engineering. The text covers all the well known methods of performing industrial measurements and problems are included at the end of each chapter. None of the instruments described is covered in great detail, since a detailed description would merely be a repetition of the manufacturer's catalogue.

The author commences by showing how one may analyze the dynamic response of an instrument which contains mass, elasticity, and friction. The remaining chapters describe specific instruments employed in the measurement of such physical quantities as temperature, pressure, displacement, velocity, acceleration, rate of flow, etc. One chapter is devoted to composition analysis, that is, various methods of spectroscopy. The last chapter briefly describes instru-

mentation as applied to an over-all plant process. Since process instrumentation is closely related to process control, some knowledge of the latter subject is required to appreciate the instrumentation problem.

From the standpoint of undergraduate work, this book would not warrant a course in itself, but could be used as a supplementary text for a course in process control.

I. GLASSMAN

BOOK NOTES

ELECTRICITY AND MAGNETISM, THEORY AND APPLICATIONS, by Norman E. Gilbert. Third edition, 569 pages, diagrams, 14 × 21 cm. New York, Macmillan Co., 1950. Price, $5.00.

Designed as a textbook in electricity and magnetism for the general or nontechnical student, this work is aimed to meet his needs for a sufficient background to understand the many present day applications of electricity and magnetism. This new edition introduces the method of using complex numbers for the treatment of vector problems, and extends the discussion of ion accelerators, nuclear chemistry and atomic energy.

MECHANICS, A TEXTBOOK FOR ENGINEERS, by James E. Boyd and Percy W. Ott. Third edition, 422 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $4.50.

Newly revised after twenty years, the present work maintains the emphasis on fundamental principles with suitable examples and problems which characterized the earlier editions. Besides the addition of new chapters and the rearrangement of others, much of the text has been rewritten. More difficult problems have been added to give the student increased opportunities to develop his skills.

INTRODUCTION TO ELECTRICITY AND OPTICS, by Nathaniel H. Frank. 440 pages, illustrations, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $5.00.

Designed as a second year textbook for students in electrical engineering or physics, this book has been extensively rewritten to expand the presentation of the physical principles underlying the subject. The mks rationalized system of units has been employed throughout the text in this edition and various additions have been made.

ELECTRICAL COMMUNICATION, by Arthur L. Albert. Third edition, 593 pages, illustrations, diagrams, 15 × 25 cm. New York, John Wiley & Sons, Inc., 1950. Price, $6.50.

In revising this text on the entire field of electrical communication, the author has made several changes. The chapters on basic fundamentals have been grouped in the front of the volume, while the more detailed treatments on telegraph, telephone and radio systems form the last part. Dial telephone systems and radio systems have been treated more fully. Review questions have been added and the problems revised. The extensive lists of references continue a valuable feature of the book.

ORGANIC CHEMISTRY, by Louis F. Fieser and Mary Fieser. Second edition, 1125 pages, illustrations, diagrams, tables, 16 × 24 cm. Boston, D. C. Heath and Co., 1950. Price, $7.50. Trade edition published by Reinhold Publishing Corp. Price, $10.00.

Many significant advances have been considered by the authors in their revision of a work first published in 1944. Newly discovered reagents, new techniques of separation and analysis, tracer studies have all made available many data which have had to be considered. In addition to the general revision of the text, new chapters have been added on Heterocyclic Compounds and Reaction Mechanisms.

PHYSICS FUNDAMENTAL PRINCIPLES FOR STUDENTS OF SCIENCE AND ENGINEERING, by George Shortley and Dudley Williams. Two volumes, 1271 pages, illustrations, diagrams, tables, 15 × 23 cm. New York, Prentice-Hall, Inc., 1950. Price, $6.00 and $7.35.

An introductory textbook in physics, designed for the college student who has had a year's work in chemistry and mathematics, high-school physics and who is taking a course in calculus. The authors have presented a "full discussion of basic ideas" and have employed an operational approach to physics. The number of systems of units employed has been kept at a minimum, those used being the ones most favored by practicing engineers. Sufficient material has been included to give the instructor an opportunity for selection in both text and problems.

HEAT AND TEMPERATURE MEASUREMENT, by Robert L. Weber. 422 pages, illustrations, diagrams, 15 × 21 cm. New York, Prentice-Hall, Inc., 1950. Price, $6.65.

Experience in the measurement of heat and temperature is valuable for students in various scientific fields, and the present text is designed to be used in a course aimed at giving the student such experience. It emphasizes experimental methods rather than thermodyanmic theory. The first part presents the physical principles which serve as a basis for thermal measurements. Worked examples and bibliographic references are features of this part. Part II describes the procedure for laboratory experiments to be used with the text. An appendix contains several tables.

INORGANIC SYNTHESES. VOLUME III. Ludwig Audrieth, Editor-in-chief. 230 pages, illustrations, 15 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $3.75.

Methods of preparation for more than fifty important inorganic compounds are included in this new volume of an important series. There are also three survey articles dealing with beryllium compounds, organosilicon compounds and the preparation of the phosphates, polyphosphates, and fluophosphates. Among the individual compounds considered, many contain fluorine, a reflection of the current interest in this element. Each synthesis contains a brief introduction, a detailed procedure, discussion of the compounds' physical and chemical properties and literature references. A cumulative index to the three volumes already published has been included.

A GERMAN-ENGLISH DICTIONARY FOR CHEMISTS, by Austin M. Patterson. Third edition, 541 pages, 13 × 18 cm. New York, John Wiley & Sons, Inc., 1950. Price, $5.00.

This is a good book that is now better—statistically more than 30 per cent better, for the number of pages and the number of entries have both been increased in approximately this ratio. There is now an estimated total of 59,000 terms, with the major additions being in the fields of chemical technology, electronics and warfare. Despite the limitation in the title to "chemists," the work is inclusive enough to be of real value to scientists in other fields. With its many useful features of the earlier editions supplemented by the greatly increased vocabulary, "Patterson" will continue to be a well-thumbed volume on any library's shelves.

A TEXT-BOOK OF INORGANIC CHEMISTRY, by J. R. Partington. Sixth edition, 996 pages, illustrations, diagrams, 14 × 22 cm. London, Macmillan and Co., Ltd., 1950. Price not given.

A standard text for advanced students, this work has again been revised and now appears in a sixth edition. Much of the more elementary matter has been omitted to permit the inclusion of new material. The sections on atomic structure and the electronic theory of valency have been extended, and considerable information has been added on the rarer metals of industrial importance. Questions for exercise and numerical problems have been omitted from this edition.

ANALYTIC GEOMETRY, by Raymond D. Douglass and Samuel D. Zeldin. 216 pages, diagrams, tables, 15 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $2.75.

This text offers a simple and concise presentation of the essentials of plane and solid analytic geometry to provide the requisite background for a course in calculus. Numerous illustrative examples and problems have been included, with answers to the latter and short tables as additional aids.

ELEMENTS OF ORDINARY DIFFERENTIAL EQUATIONS, by Michael Golomb and Merrill Shanks. 356 pages, diagrams, 15 × 23 cm. New York, McGraw-Hill Book Company, Inc., 1950. Price, $3.50.

Intended for use in a first course in ordinary differential equations and with students who have had only elementary calculus, this text has been designed to appeal to students majoring in engineering, science or mathematics. Some features are early emphasis on geometric and numerical methods, use of the existence theorems in development of techniques, use of Green's functions, and an elementary but precise treatment of power series. Difficult problems or those extending the theory are marked with a star. Answers are given for nearly all problems.

THE CONDENSED CHEMICAL DICTIONARY, by Francis M. Turner, editorial director. Fourth edition, 726 pages, 15 × 23 cm. New York, Reinhold Publishing Corp., 1950. Price, $10.00.

This new edition of a useful work has been extensively revised. Many new terms have been included, particularly more trade and brand names. Even common items show evidence of the revision, both in phraseology and in new data. A new feature of this volume is the indication by means of a number key of the manufacturers of the various trade and brand name products which will prove particularly useful. Since some entire entries have been omitted in this edition and others shortened, the previous edition will still have some usefulness.

ENGINEERING ECONOMY, by H. G. Thuesen. 501 pages, 15 × 22 cm. New York, Prentice-Hall, Inc., 1950. Price, $5.00.

A textbook which presents various economic features which should be considered by an engineer to insure successful handling of his problems. Emphasis is placed on the necessity of arriving at present decisions that will be fruitful in the future. Concrete methods of analyses are given and numerous problems of a practical nature are offered for the students' practice.

ANNUAL REPORTS ON THE PROGRESS OF CHEMISTRY FOR 1949, issued by the Chemical Society. Vol. 46. 333 pages, 14 × 23 cm. London, Chemical Society, 1950. Price, 25 s.

The present volume in this long-established series is in a sense a transition volume as it marks the beginning of a return to a policy of covering only the literature of one year instead of presenting comprehensive essays on selected topics covering a period of years. Accordingly, the present volume in large measure attempts to cover fields which have not been treated for several years, at the same time reviewing the past year. Thus there is some unevenness in fullness of treatment, which is probably unavoidable under the circumstances.

The major groupings are general and physical chemistry, crystallography, inorganic chemistry, organic chemistry, biochemistry and analytical chemistry.

In the first section there are ten brief summaries of various topics, with two more detailed papers on Magnetochemistry by R. C. Pink and The Pauling Hypothesis by W. Hume-Rothery. In the field of crystallography there is a long article on crystal chemistry by (Mrs.) D. Crowfoot Hodgkin. In organic chemistry the longest paper is on Theoretical Organic Chemistry by E. A. Braude, in which the emphasis has been placed on the mechanism of organic reactions.

Always a useful review volume, the full reversion to the former treatment will be awaited with interest.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

Production of Liver Tumors in Rats on a Nutritionally Adequate Diet for Growth by 3′-Methyl-4-dimethylaminoazobenzene.—J. O. ELY AND M. H. ROSS. The diets that have been employed by investigators in the experimental production of liver cancer in rats with certain oil-soluble dyes, such as 4-dimethylaminoazobenzene, generally have been poor nutritionally in that they restricted the growth of the experimental

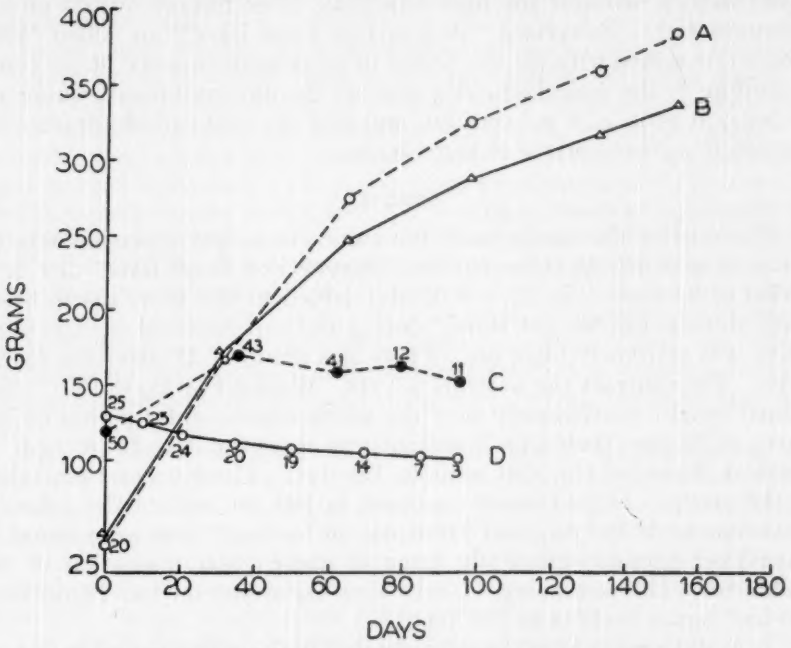

FIG. 1.—Growth of rats fed Wayne Fox Food Blox, 12 per cent casein basal diet without and with 3′-Me-4-DAB.

A—Wayne Fox Food Blox diet.
B—Wayne Fox Food Blox diet plus 3′-Me-4-DAB.
C—Casein basal diet.
D—Casein basal diet plus 3′-Me-4-DAB.

animals. Sasaki and Yoshida (4) and Kinoshita (1), the first investigators in this field, incorporated the dyes in a diet of rice flour supplemented with carrot, or with meat scraps, salts and corn oil or olive oil. When various supplements as liver, yeast, or casein were added to the rice flour basal diets containing the dyes, tumors did not develop during experimental periods of over 250 days (2, 5). Even in

the experimental production of liver tumors with the more highly carcinogenic dyes, such as 3′-methyl-4-dimethylaminoazobenzene (3′-Me-4-DAB), which according to Price *et al* (3) is twice as carcinogenic as 4-dimethylaminoazobenzene, nutritionally poor basal diets are generally used. The original basal diet of rice flour as the main ingredient employed by the Japanese workers, however, has been largely replaced by semisynthetic diets containing casein instead of rice flour. Mortality of experimental animals on a typical basal diet with 3′-Me-4-DAB is very great as will be indicated later.

It was considered of interest to determine if this dye, 3′-Me-4-DAB, would induce, without the high mortality, liver tumors in rats on an adequate diet. Pulverized "Wayne Fox Food Blox," an Allied Mills product, was used with the dye added to a concentration of 0.06 per cent. According to the manufacturer's analysis the diet contained minima of 25 per cent protein, 4 per cent fat, and 45.5 per cent carbohydrate, and contained a great variety of ingredients.

RESULTS

The rats fed the casein basal diet gained in weight approximately 10 per cent as much as those fed the "Wayne Fox Food Blox" diet in a period of 94 days (Fig. 1). With dye added to this diet (casein basal diet) animals lost weight slowly during the experimental period; mortality was extremely high, only 3 rats of a group of 25 surviving for 94 days. For contrast the animals fed the "Wayne Fox Food Blox" diet gained weight continuously over the whole experimental period of 222 days; at 98 days their gain in weight was approximately 85 per cent as great as those fed this diet without the dye. There was no mortality in the group. Liver tumors appeared in 100 per cent of the animals. Between the 162nd day and 216th day of feeding 9 rats were found to have liver tumors sufficiently large to cause great distension of the abdomen. The remaining 11 rats were autopsied on the 222nd day. All had tumor nodules in the liver.

It would appear from these results that in the experimental induction of liver tumors by 3′-Me-4-DAB, nutritionally adequate basal diets may be used; mortality of the experimental animals is thus reduced to a minimum and a high incidence of tumors results.

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CURRENT TOPICS

Braille Reader.—The burden of bulk which makes Braille literature cumbersome and difficult to handle is considerably reduced by a new machine called a Braille Reader developed by the International Business Machines Corp. Although the development is in an experimental stage, the machine was shown and demonstrated at The Franklin Institute during a meeting of the American Association of Instructors for the Blind.

The reduction of 75 per cent of the physical bulk is made possible by the use of punched tapes rather than standard Braille sheets embossed on only one side of the page. The use of punched tapes also makes possible the more rapid and more economical reproduction of Braille material in large quantities from a single master tape.

In the Braille Reader, the tape, punched in the Braille code, passes over a sensing mechanism which controls an electro-mechanical device which, in turn, sets up the Braille by means of movable pins set in an endless plastic belt.

As the belt passes across the reading station, an area approximately 10 inches long and one inch wide, the pins remain upright reproducing the Braille signs punched in the tape. The operator, sitting at the compact machine, can read at any point in the reading area. This makes it possible for a reader who may have missed a letter, word or sign to pick it up by stopping the machine at once and tracing the sign in the reading station. The Braille Reader's speed ranges from approximately 50 to 150 words a minute; its slowest speed being suited for the beginner in Braille. A speed regulator permits each reader to adjust the machine to the speed of his individual ability. Both the reading speed regulator and the on-off switch are embodied in one control permitting the reader to keep one hand at the reading station during the entire operating period.

Once the pins have passed the reading station they are depressed to the under side of the plastic belt. They are then ready to be set in new Braille positions. It is a characteristic of plastic to form, when perforated, an "hour-glass" shape within the hole. It is the utilization of this characteristic in the IBM Braille Reader's plastic belt that makes it possible for the pins to remain securely in place while being read under normal finger pressure and to retain their upright position along the entire reading area.

The Braille Reader is the result of many years of research by the late James W. Bryce who, as Chief Consultant of IBM's engineering and development laboratories, and dean of the company's technical staff, had been at work on this machine at the time of his death in March 1949.

An important feature of the IBM Braille Reader is that it requires no adjustment on the part of the reader to its mode of operation other than the subjective one of becoming accustomed to the Braille moving beneath the finger rather than the finger moving across the embossed Braille. Those who have used the IBM Braille Reader say they were able to adjust to this feature within ten minutes.

Bonded Bi-metallic Stator Blades.—Bi-metallic gas turbine stator blades, having aluminum alloy roots for easy machining, are under study in England according to an announcement of Wellworthy Pistons, Ltd., holder of the

British license for the molecular bonding of aluminum to steel by means of the Al-Fin process. Primary object of the study is the development of a rapid, economical method of manufacturing compressor stator assemblies. Wellworthy is licensed by the Al-Fin Division of the Fairchild Engine and Airplane Corporation, developer of the patented molecular bonding process.

Bright drawn steel strip is used for the airfoil section. The strip, which can be obtained to extremely close dimensions from the mills, is cropped to length and—by means of the Al-Fin process—an aluminum alloy root is bonded to it. The alloy used is selected for its corrosion resistance and strength. The immediate advantage of the method is that it provides a means of manufacturing compressor stator blades with a root section that is readily machineable—hence less expensive.

In addition, development is foreseen of compressor stator blade units in segments containing a number of blades or even complete stage rings with blade sections made integral by bonding in of the drawn airfoil strip steel. Cost and weight advantages are foreseen as resulting from such construction.

Extensive testing has proved the strength of the Al-Fin aluminum-to-steel bond. The Al-Fin bond has a tensile strength of approximately 15,000 psi. Tensile tests at elevated temperatures show that the bond retains its strength at temperatures at least as high as the 600 to 700 degrees F encountered in compressors. The bond is vibration proof and is leakproof to hot oil and diatomic gases under high pressures.

Pure aluminum and any of the common casting alloys such as 43, 142, 195, 355, 356, Almag 35, Frontier 40E, etc., which have much higher tensile strength than pure aluminum, can be bonded to cast iron, "Ni-resist", carbon, alloy, stainless, inconel and the nimonic series. It is possible to heat treat after bonding, in some cases, to obtain desired physical properties.

Besides applications such as that being investigated by Wellworthy, the Al-Fin process is being used extensively for such things as aluminum-finned steel cylinder barrels, steel-backed aluminum bearings, bi-metallic pistons with ferrous ring carriers, aluminum gears with bonded-in-steel hubs, lightweight aluminum housings with steel or iron bearing retainers, bi-metallic automotive brake drums, etc.

A compact, inexpensive supersonic wind-tunnel simulator, which uses a closed system of fast-moving water at controlled speeds in place of air, has been developed for test and demonstration purposes by the Navy's Special Devices Center, Sands Point, L. I. Capable of roughly accurate mathematical calculation of supersonic effects, the table-top device simulates work done in scarce and costly tunnels, though it could not be used for final studies.

A helicopter airport atop a waterfront warehouse is being maintained at the Seattle Port of Embarkation to expedite delivery to the Port of high-priority military papers from inland Forts Lewis and Worden. Regular runs to the rooftop airport, on a five-story concrete warehouse at the piers, make delivery a matter of minutes instead of several hours. Standard airport appurtenances in use include a wind sock, white guide markings, and brightly colored flags to show obstructions.

THE FRANKLIN INSTITUTE

exists today because of the faith and generosity of the men and women who for 125 years have given time and money to its support.

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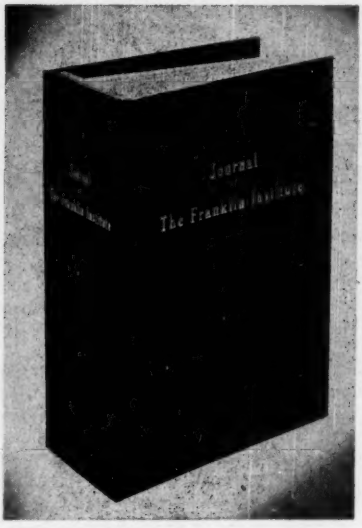

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The Elliott Cresson Medal (1848—Gold Medal).—This medal is awarded for discovery or original research, adding to the sum of human knowledge, irrespective of commercial value; leading and practical utilizations of discovery; and invention, methods or products embodying substantial elements of leadership in their respective classes, or unusual skill or perfection in workmanship.

The Howard N. Potts Medal (1906—Gold Medal).—This medal is awarded for distinguished work in science or the arts; important development of previous basic discoveries; inventions or products of superior excellence or utilizing important principles.

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The Louis E. Levy Medal (1923—Gold Medal).—This medal is awarded to the author of a paper of especial merit, published in the JOURNAL OF THE FRANKLIN INSTITUTE, preference being given to one describing the author's experimental and theoretical researches in a subject of fundamental importance.

The George R. Henderson Medal (1924—Gold Medal).—This medal is awarded for meritorious inventions or discoveries in the field of Railway Engineering.

The Walton Clark Medal (1926—Gold Medal).—This medal is awarded to the "author of the most notable advance in knowledge or improvement in apparatus, or in method concerning the science or the art of gas manufacture or distribution or utilization in the production of illumination, or of heat, or of power."

The Frank P. Brown Medal (1938—Silver Medal).—This medal is awarded to inventors for discoveries and inventions involving meritorious improvements in the building and allied industries.

The Newcomen Medal (1943—Gold Medal).—This medal is awarded, not oftener than once in three years, for achievement in the field of Steam.

The Francis J. Clamer Medal (1943—Silver Medal).—This medal is awarded at least once in five years for meritorious achievement in the field of Metallurgy.

The Stuart Ballantine Medal (1946—Gold Medal).—This medal is to be awarded in recognition of outstanding achievement in the fields of Communication and Reconnaissance which employ electromagnetic radiation.

The William M. Vermilye Medal (1937—Bronze Medal).—This medal is awarded not oftener than biennially in recognition of outstanding contribution in the field of Industrial Management.

The Certificate of Merit (1882).—A Certificate of Merit is awarded to persons adjudged worthy thereof for meritorious inventions, discoveries or improvements in physical processes or devices.

The Boyden Premium.—Uriah A. Boyden, Esq., of Boston, Mass., has deposited with The Franklin Institute the sum of one thousand dollars, to be awarded as premium to "any resident of North America who shall determine by experiment whether all rays of light and other physical rays are or are not transmitted with the same velocity."

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(Revised to April, 1948.)

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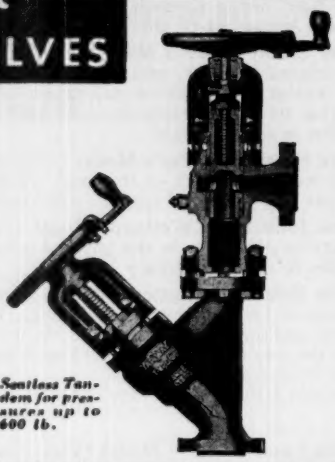

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